Warm up
Solve each equation

$x = 4, +3$

1. $x^2 - 7x + 12 = 0$

$(x-4)(x-3)$

2. $4x^2 - x = 0$

$x(4x-1)=0$

$x=0$ $x=\frac{1}{4}$

3. $2x^2 - 7x - 15 = 0$

-30

-10 3

$(2x-10)(2x+3)$

$(x-5)(2x+3)$

$x=5$ $x=-\frac{3}{2}$

4. $x^2 - 4x - 2 = 0$

$x = \frac{4 \pm \sqrt{16-4(1)(-2)}}{2}$

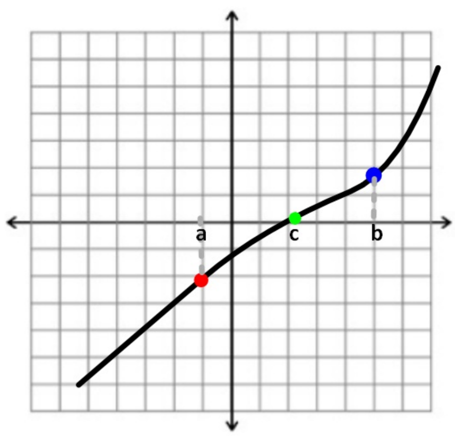
$x = \frac{4 \pm \sqrt{24}}{2}$ 4 6

$x = \frac{4 \pm 2\sqrt{6}}{2}$

$\boxed{x = 2 \pm \sqrt{6}}$

Intermediate Value Theorem In its essence it ensures that if we have a continuous function with a closed interval [a,b], then every y-value between f(a) and f(b) is guaranteed to be a part of the range of the function.

an interpretive simplification is:
--> a sign change in the range implies a real zero exists somewhere between the two values of the range.



If $[a,b]$ and
$f(a) = -$ &
$f(b) = +$
then a zero exists between a & b

2.4/2.5

Objective: Find Zeros by dividing polynomials
Find the remainder of a root
Determine whether a binomial is a factor of a polynomial

Divide using long divison: $$\frac{3x^3 + 5x^2 + 8x + 7}{3x + 2}$$

It doesn't make sense to use long division on this problem...it is appropriate, but it isn't the most efficient way. So I would use synthetic division.

Long division MUST be used when the divisor has degree 2 or higher.

$$3x + 2 = 0$$
$$3x = -2$$
$$x = -\frac{2}{3}$$

$$\begin{array}{r|rrrr} -\frac{2}{3} & 3 & 5 & 8 & 7 \\ & \downarrow & -2 & -2 & -4 \\ \hline & 3 & 3 & 6 & \boxed{3} \end{array}$$

$$3x^2 + 3x + 6 + \frac{3}{3x+2}$$

Using Polynomial Long Division

MUST use long as divisor deg > 2

Use long division to find the quotient and remainder when $2x^4 - x^3 - 2$ is divided by $2x^2 + x + 1$.

$$\boxed{x^2 - x + \frac{x-2}{2x^2+x+1}}$$

$$\boxed{2x^2} + x + 1 \,\big)\, \boxed{2x^4} - x^3 + 0x^2 + 0x - 2$$

$$\cancel{-2x^4} + x^3 + x^2$$

$$\boxed{-2x^3} - x^2 + 0x - 2$$

$$\cancel{+2x^3} \; \cancel{+x^2} + x$$

$$\boxed{x} - 2$$

power out > power in so stop

:)

Using Synthetic Division

~ only w/ a divisor of degree 1

Divide $2x^3 - 3x^2 - 5x - 12$ by $x - 3$ using synthetic division and write a summary statement in fraction form.

① set divisor = 0 & solve

$x - 3 = 0$

$x = 3$

② set up & ÷

add

mult

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & \boxed{0} \\ & x^2 & x^1 & c & R \end{array}$$

$2x^2 + 3x + 4$

Divide Synthetically: $(x^4 - 3x^3 + 5x - 6)(x + 2)^{-1}$

$x + 2 = 0$

$x = -2$

$$\begin{array}{r|rrrrr} -2 & 1 & -3 & 0 & 5 & -6 \\ \hline \end{array}$$

power of -1 means divide

THEOREM Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Determine the value of the remainder using the Remainder Theorem

$$f(x) = 3x^2 + 7x - 20 \text{ is divided by } x + 4.$$

1. set the divisor = 0 and solve
2. substitute that value into the function and simplify
3. the result is the same remainder you would obtain if you long/synthetically divide.

$$f(-4) = 3(-4)^2 + 7(-4) - 20$$

$$f(-4) = 0$$

Thus, the remainder is zero.

THEOREM Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

$x - 2;\ \overset{f(x)}{x^3 - 3x - 2}$

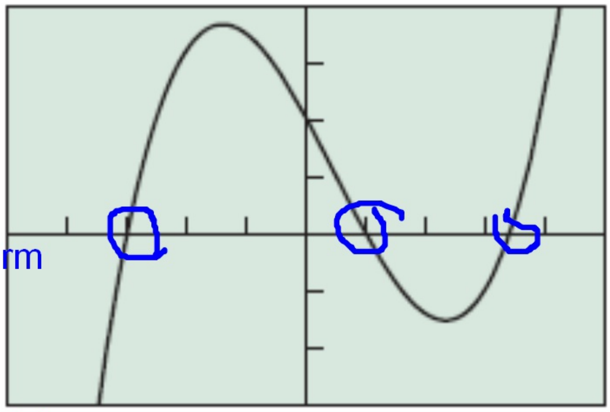
1. set the divisor = 0 and solve
2. substitute that value into the function and simplify
3. if the result is zero specifically, then the divisor is a factor of the dividend

$$f(2) = (2)^3 - 3(2) - 2$$
$$= 8 - 6 - 2$$
$$f(2) = 0$$

Thus, x-2 is a factor of f(x).

In Exercises 25 and 26, use the graph to guess possible linear factors of $f(x)$. Then completely factor $f(x)$ with the aid of synthetic division.

$$f(x) = 5x^3 - 7x^2 - 49x + 51$$



[–5, 5] by [–75, 100]

guess??? The zeros give us the factors. Here's how....

1. state the zeros
2. "unsolve" the zeros for factor form by moving the values to the left and making it equal to zero.

$x = -3$ $x = 1$ $x = 3.4$

$(x+3)$ $(x-1)$ $(x-3.4)$