Warm up
Solve each equation

$x = 4, +3$

1. $x^2 - 7x + 12 = 0$

$(x-4)(x-3)$

2. $4x^2 - x = 0$

$x(4x-1)=0$

$x = 0$ $\quad$ $x = \frac{1}{4}$

3. $2x^2 - 7x - 15 = 0$

-30

$-10 \quad 3$

$(2x-10)(2x+3)$

$(x-5)(2x+3)$

$x = 5 \quad x = -\frac{3}{2}$

4. $x^2 - 4x - 2 = 0$

$x = \frac{4 \pm \sqrt{16-4(1)(-2)}}{2}$

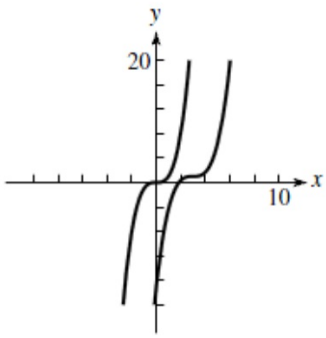
$x = \frac{4 \pm \sqrt{24}}{2}$ $\quad$ (4 · 6)

$x = \frac{4 \pm 2\sqrt{6}}{2}$

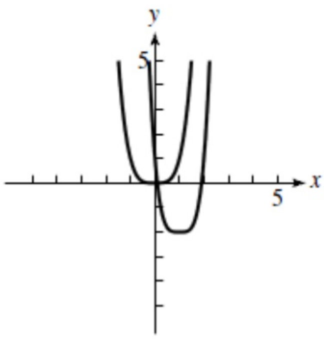
$x = 2 \pm \sqrt{6}$

Homework Section 2.3

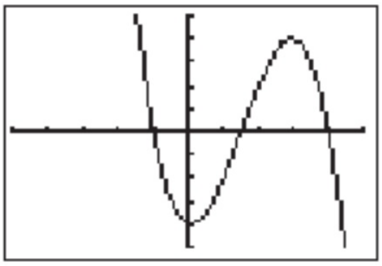
4. Shift $y = x^3$ to the right by 3 units, vertically shrink by $\frac{2}{3}$, and vertically shift up 1 unit. y-intercept: $(0, -17)$



6. Shift $y = x^4$ to the right 1 unit, vertically stretch by 3, and vertically shift down 2 units. y-intercept: $(0, 1)$



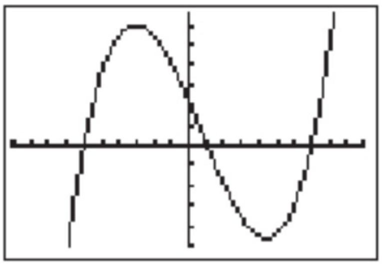
18.



$[-5, 5]$ by $[-15, 15]$

$\lim_{x \to \infty} f(x) = -\infty; \lim_{x \to -\infty} f(x) = \infty$

20.



$[-10, 10]$ by $[-100, 130]$

$\lim_{x \to \infty} f(x) = \infty; \lim_{x \to -\infty} f(x) = -\infty$

26. $f(x) = -x^3 + 7x^2 - 4x + 3$ $-\infty, \infty$

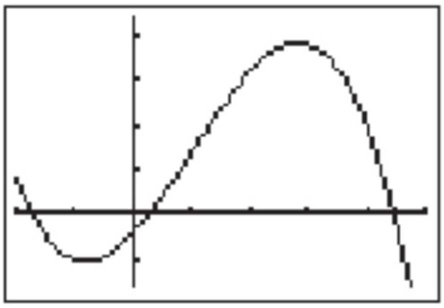
34. $f(x) = 3x^2 + 4x - 4$ -2 and $\frac{2}{3}$

36. $f(x) = x^3 - 25x$ 0, -5, and 5

38. $f(x) = 5x^3 - 5x^2 - 10x$

0, -1, and 2

44.



$[-2, 5]$ by $[-8, 22]$

-1.73, 0.26, 4.47

52. $f(x) = x^3 - 4x^2 - 44x + 96$ -6, 2, and 8

54. $f(x) = (x + 2)(x - 3)(x + 5) = x^3 + 4x^2 - 11x - 30$

56. $1, 1 + \sqrt{2}, 1 - \sqrt{2}$

56. $f(x) = x^3 - 3x^2 + x + 1$

In Exercises 71 and 72, solve the problem without using a calculator.

71. Multiple Choice What is the y-intercept of the graph of $f(x) = 2(x - 1)^3 + 5$? C

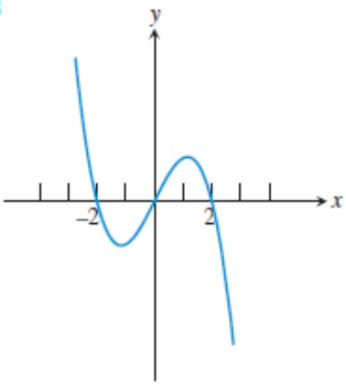
(A) 7 **(B)** 5 **(C)** 3 **(D)** 2 **(E)** 1

72. Multiple Choice What is the multiplicity of the zero $x = 2$ in $f(x) = (x - 2)^2(x + 2)^3(x + 3)^7$? B

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

In Exercises 73 and 74, which of the specified functions might have the given graph?

73. Multiple Choice B



(A) $f(x) = -x(x + 2)(2 - x)$

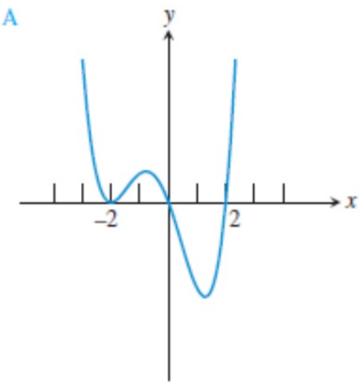
(B) $f(x) = -x(x + 2)(x - 2)$

(C) $f(x) = -x^2(x + 2)(x - 2)$

(D) $f(x) = -x(x + 2)^2(x - 2)$

(E) $f(x) = -x(x + 2)(x - 2)^2$

74. Multiple Choice A



(A) $f(x) = x(x + 2)^2(x - 2)$

(B) $f(x) = x(x + 2)^2(2 - x)$

(C) $f(x) = x^2(x + 2)(x - 2)$

(D) $f(x) = x(x + 2)(x - 2)^2$

(E) $f(x) = x^2(x + 2)(x - 2)^2$

2.4/2.5

Objective: Find Zeros by dividing polynomials
Find the remainder of a root
Determine whether a binomial is a factor of a polynomial

Divide using long divison:

$$\frac{3x^3 + 5x^2 + 8x + 7}{3x + 2}$$

Using Polynomial Long Division

Use long division to find the quotient and remainder when $2x^4 - x^3 - 2$ is divided by $2x^2 + x + 1$.

$$x^2 - x + \frac{x-2}{2x^2+x+1}$$

$$2x^2 + x + 1 \,\overline{)\, 2x^4 - x^3 + 0x^2 + 0x - 2}$$

$$-2x^4 + x^3 + x^2$$

$$-2x^3 - x^2 + 0x - 2$$

$$+2x^3 + x^2 + x$$

$$x - 2$$

power out > power in so stop :)

Using Synthetic Division

~only w/ a divisor of degree 1

Divide $2x^3 - 3x^2 - 5x - 12$ by $x - 3$ using synthetic division and write a summary statement in fraction form.

① set divisor = 0 & solve

$x - 3 = 0$

$x = 3$

② set up & ÷

add

3	2	-3	-5	-12
	↓	6	9	12
mult	2	3	4	0
	x^2	x^1	C	R

$2x^2 + 3x + 4$

Divide Synthetically: $(x^4 - 3x^3 + 5x - 6) \div (x + 2)$

$x + 2 = 0$

$x = -2$

-2	1	-3	0	5	-6

THEOREM Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

EXAMPLE 2 Using the Remainder Theorem

Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by $x + 4$.

THEOREM Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

In Exercises 19–24, use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

$x - 2;\ x^3 - 3x - 2$

Fundamental Connections for Polynomial Functions

For a polynomial function f and a real number k, the following statements are equivalent:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$.
2. k is a zero of the function f.
3. k is an x-intercept of the graph of $y = f(x)$.
4. $x - k$ is a factor of $f(x)$.

THEOREM Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0,$$

with every coefficient an integer and $a_0 \neq 0$. If $x = p/q$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0, and
- q is an integer factor of the leading coefficient a_n.

Finding the Rational Zeros

In Exercises 33–36, use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

$f(x) = (3x^3 - 7x^2)(+ 6x - 14)$

$x^2(3x-7)+2(3x-7)$ factorable ∴ use that solve

$f(x) = (3x-7)(x^2+2)$

RRT

$P = 14$ $\pm1, \pm2, \pm7, \pm14$

$Q = 3$ $\pm1, \pm3$

$\frac{P}{Q} = \left\{\pm1, \pm\frac{1}{3}, \pm2, \pm\frac{2}{3}, \pm7, \pm\frac{7}{3}, \pm14, \pm\frac{14}{3}\right\}$

List of possible rational zeros

$\frac{7}{3}$ | 3 -7 6 -14
↓ 7 0 14
3 0 6 | 0

x^2 x^1 C R

$3 \cdot x = \frac{7}{3} \cdot 3$

$3x = 7$

$-7 \quad -7$

$3x - 7 = 0$

$(3x-7)(3x^2+6)$

$3(3x-7)(x^2+2)$

factorization

Solutions

$x = \frac{7}{3}$

$x^2+2=0$

$\sqrt{x^2} = \sqrt{-2}$

$x = \pm\sqrt{-2}$

$x = \pm i\sqrt{2}$

Finding the Real Zeros of a Polynomial Function

Find all of the real zeros of $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$.

List of possible Rational zeros: $\{\pm 8, \pm 4, \pm 2, \pm 1, \pm\frac{1}{2}\}$

4	2	-7	-8	14	8
	↓	8	4	-16	-8
$-\frac{1}{2}$	2	1	-4	-2	0
	↓	-1	0	2	
	2	0	-4	0	
	x^2	x^1	C	R	

factorization

$(x-4)(2x+1)(2x^2-4)$

$2(x-4)(2x+1)(x^2-2)$

$2 \cdot x = -\frac{1}{2} \cdot 2$

$2x = -1$

$+1 \quad +1$

$(2x+1) = 0$

Roots/zeros/solns

$2 \neq 0$ | $x-4=0$, $x=4$ | $2x+1=0$, $x=-\frac{1}{2}$ | $x^2-2=0$, $\sqrt{x^2}=\sqrt{2}$, $x=\pm\sqrt{2}$

2.5 Complex Zeros

Exploring Fundamental Polynomial Connections

Write the polynomial function in standard form, and identify the zeros of the function and the x-intercepts of its graph.

(a) $f(x) = (x - 2i)(x + 2i)$

$f(x) = x^2 + 2ix - 2ix - 4i^2$

$f(x) = x^2 - 4i^2$

$f(x) = x^2 + 4$

Zeros

$x - 2i = 0$

$x = 2i$

$x + 2i = 0$

$x = -2i$

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

x-int: none

→ must be ℝ

(b) $f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

$f(x) = (x-5)(x^2 - 2i^2)$

$f(x) = (x-5)(x^2+2)$

$f(x) = x^3 + 2x - 5x^2 - 10$

$f(x) = x^3 - 5x^2 + 2x - 10$

zeros: $x = 5$ $x = \pm\sqrt{2}\,i$

x-int: $x = 5$

Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include -3, 4, and $2 - i$. $(2+i)$

$x = -3$ $x = 4$ $x = 2 - i$ $x = 2 + i$

$(x+3)(x-4)(x-2+i)(x-2-i)$

$(x^2 - x - 12)\ (x^2 - 2x - ix - 2x + 4 + 2i + ix - 2i - i^2)$

$(x^2 - x - 12)\ (x^2 - 4x + 4 + 1)$

$(x^2 - x - 12)\ (x^2 - 4x + 5)$ $i^2 = -1$

$x^4 - 4x^3 + 5x^2 - x^3 + 4x^2 - 5x - 12x^2 + 48x - 60$

$x^4 - 5x^3 - 3x^2 + 43x - 60$