Welcome to Pre-Calculus Honors with Mrs. Higgins

Please grab a book from the pile, find the seat with your name on it. Then in the front cover of your book, write in PEN your legal name and under ISSUED, write K.HIGGINS. Write the name you prefer to be called under your name on the notecard taped to your desk. On the back, please answer the following questions:

1. *your email and cell phone number*
2. *your teacher and grade for Algebra II Honors*
3. *your favorite subject in school*
4. *one interesting fact about yourself*
5. *your extracurricular activities*

1.2 Functions and Their Graphs

Function Notation:

- Determine if a function:

  Algebraically: if y is to an even power or inside absolute value bars, then not a function

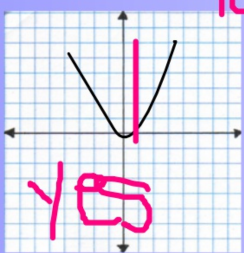
  Graphically: use the verticle line test

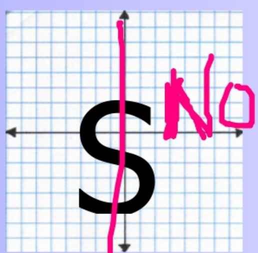
  Ex Determine if the given information represents a function in terms of x.

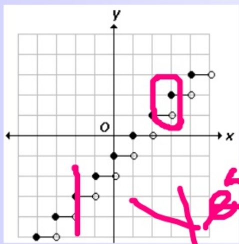
A. $x^2 + y - 3 = 2y + 7$ Yes

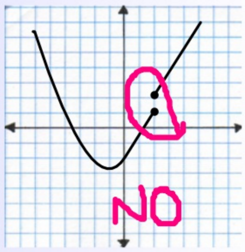
B. $|y + 2| = 5$ NO

C. $8 - y^2 + 2x = 9$ NO

D. 

E. 

F. 

G. 

Domain and Range

- Domain: set of all possible x-values
- Range: set of all possible y-values

~We need to be able to do domain from both a graph and an equation.
~We primarily do range from a graph. (for now!)
~We will use *interval notation*.

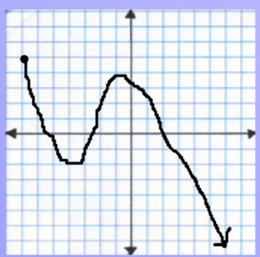
*we use a square bracket to indicate "equal to"; including [ ]

*we use parenthesis to indicate "up to but not equal to"; excluding ( )

***caution: do not look at domain/range values in parenthesis as an ordered pair*

*"all real numbers" is written as: $(-\infty, \infty)$

Ex State the domain and range using interval notation.



$D: [-7, \infty)$

$R: (-\infty, 5]$

Domain from an Equation

~two things that mess up domain

1. variable(s) under even-indexed radicals
2. variable(s) in the denominator

~if either of these is present, the domain will be *restricted---not "all real numbers"*

Ex State the domain in interval notation. Do not use a calculator!

A. $f(x) = \sqrt{3x-12}$

$3x-12=0$

$3x=12$

$x=4$

D: $[4,\infty)$

B. $f(x) = \dfrac{7x+4x^2-11}{3x^2-7x-6}$ — who cares!

$3x^2-7x-6=0$

$(3x-9)(3x+2)=0$

$(x-3)(3x+2)=0$

$x=3 \quad x=-2/3$

No sign chart, cause no $\sqrt{\ }$

D: $(-\infty,-\frac{2}{3}) \cup (-\frac{2}{3},3)$

$\cup (3,\infty)$

Domain from an Equation

~two things that mess up domain

1. variable(s) under even-indexed radicals
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Ex State the domain in interval notation. Do not use a calculator!

A. $f(x) = \sqrt{3x-12}$

① $3x-12=0$

$x=4$

$-$ $+$ $3x-12$

0 4 5

D: $[4, \infty)$

B. $f(x) = \dfrac{7x+4x^2-11}{3x^2-7x-6}$

$3x^2-7x-6=0$

M: -18, A: -7; -9, 2

$(3x-9)(3x+2)=0$

$(x-3)(3x+2)=0$

$x \neq 3 \quad x \neq -\frac{2}{3}$

$-\frac{2}{3}$ 3

D: $(-\infty, -\frac{2}{3}) \cup (-\frac{2}{3}, 3) \cup (3, \infty)$

9. $\dfrac{\sqrt{x+2}}{\sqrt{3-x}}$

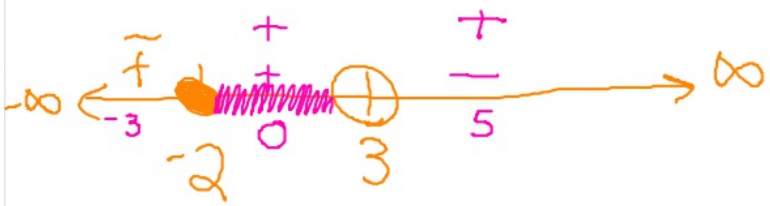
$(\sqrt{3-x})^2 = (0)^2$

$3 - x = 0$

$x \neq 3$

$x + 2 = 0$

$x = -2$



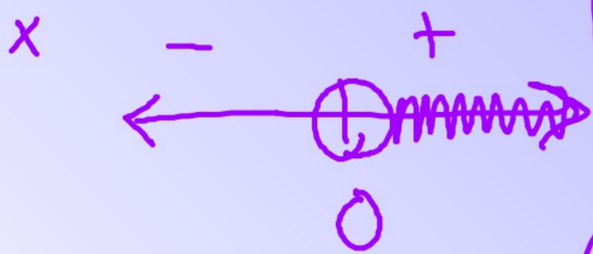
D: $[-2, 3)$

C. $f(x) = \dfrac{7}{\sqrt{x}}$

you try!

$x \neq 0$

& $x \neq$ neg #



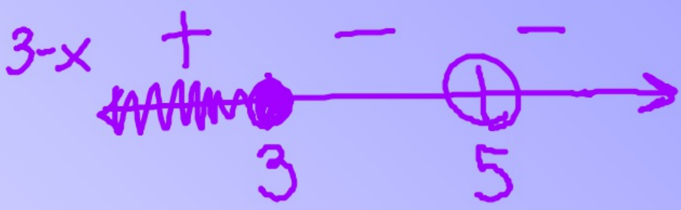
D: $(0, \infty)$

D. $f(x) = \dfrac{\sqrt{3-x}}{x-5}$

$x \neq 5$

$3 - x = 0$

$x = 3$



D: $(-\infty, 3]$

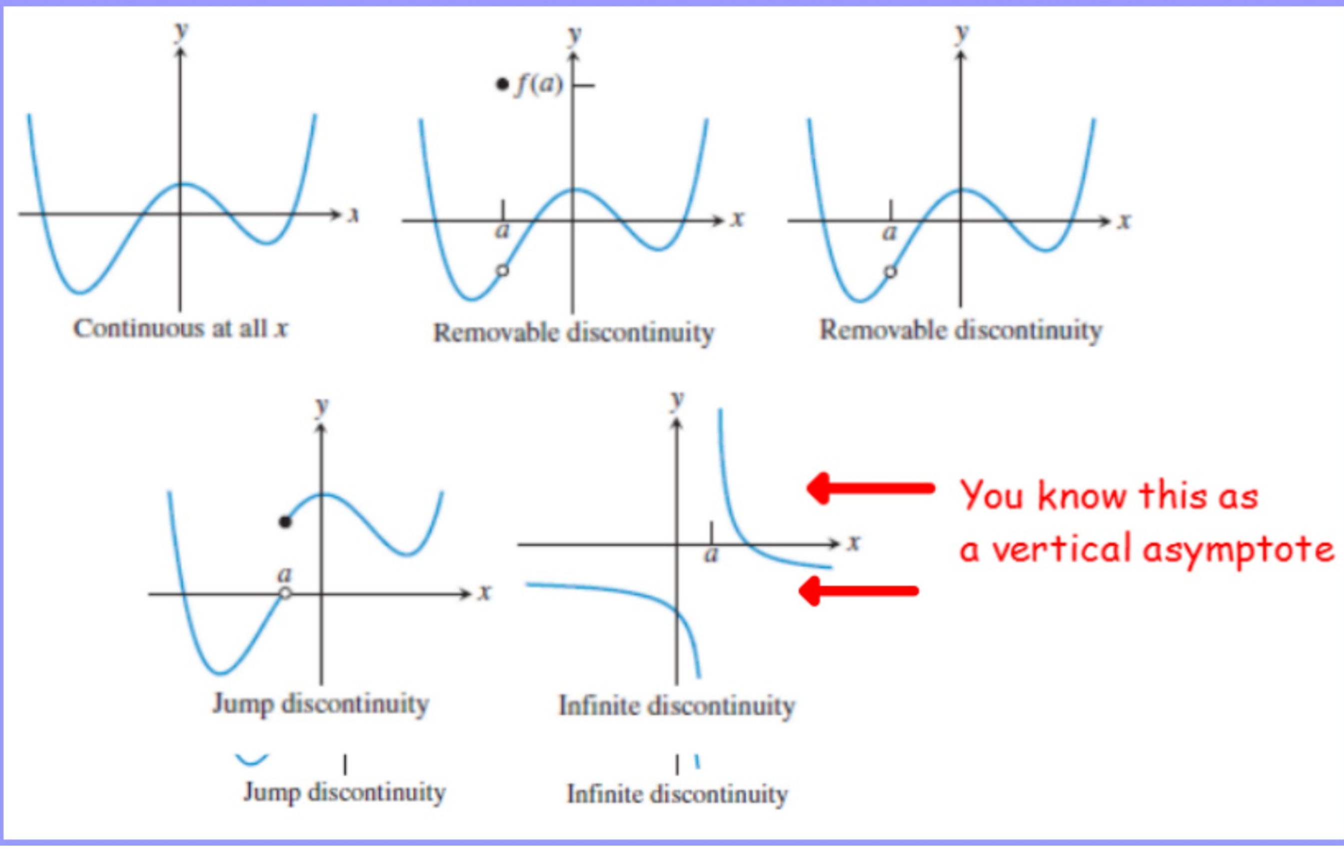
* only sign tested into the $\sqrt{\ }$ part

other part just can't be zero

Continuity

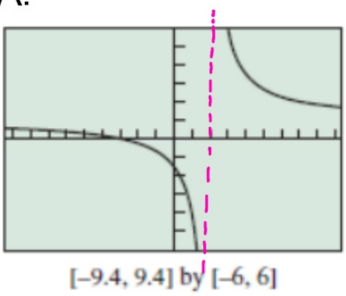
~a ***continuous*** function can be traced/drawn without picking up the pencil

~a ***discontinuous*** function has a hole, a gap or a jump of some kind



Ex Using the graphs, label each function as continuous or discontinuous.
If discontinuous, state the type of discontinuity and where it occurs.

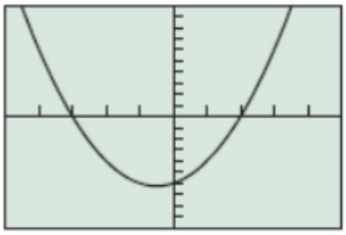
A.



[–9.4, 9.4] by [–6, 6]

$f(x) = \frac{x+3}{x-2}$

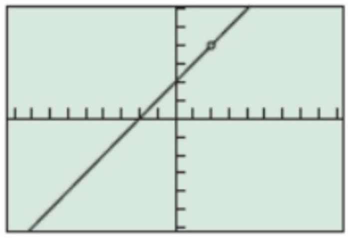
B.



[–5, 5] by [–10, 10]

$g(x) = (x+3)(x-2)$

C.



[–9.4, 9.4] by [–6.2, 6.2]

$\cancel{(x-2)}(x+2)$

$h(x) = \frac{x^2-4}{\cancel{x-2}}$

$\frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x+7)}$

remov. @ $x=-2$

nonremov. @ $x=-7$

D: $(-\infty,-7) \cup (-7,-2) \cup (-2,\infty)$

Ex

given: $f(x) = \begin{cases} kx-1, & x \geq 2 \\ x+2, & x < 2 \end{cases}$

state the value of k that makes the function continuous.

$$kx - 1 = x + 2$$

$$k(2) - 1 = (2) + 2$$

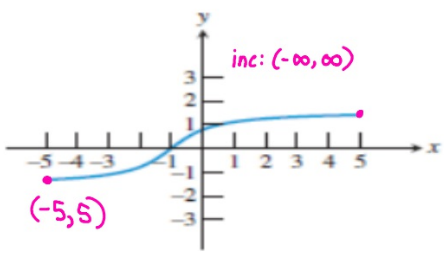
$$2k - 1 = 4$$

$$2k = 5$$

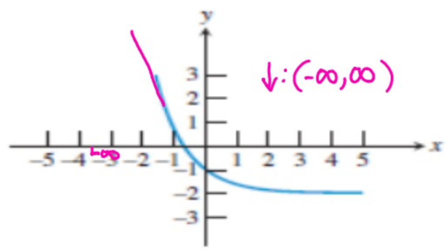
$$k = \frac{5}{2}$$

Increasing, Decreasing & Constant

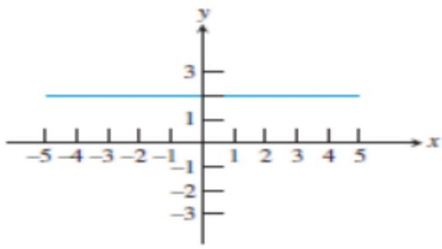
~describes the behavior of the y-values in terms of a zone of the x-axis
~use interval notation to indicate intervals
~only x-values are a part of the interval notation
~use parenthesis only



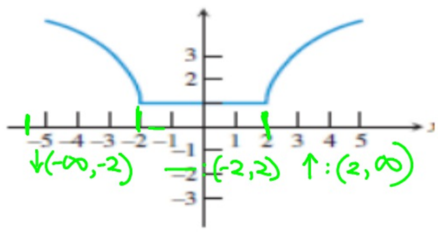
Increasing



Decreasing

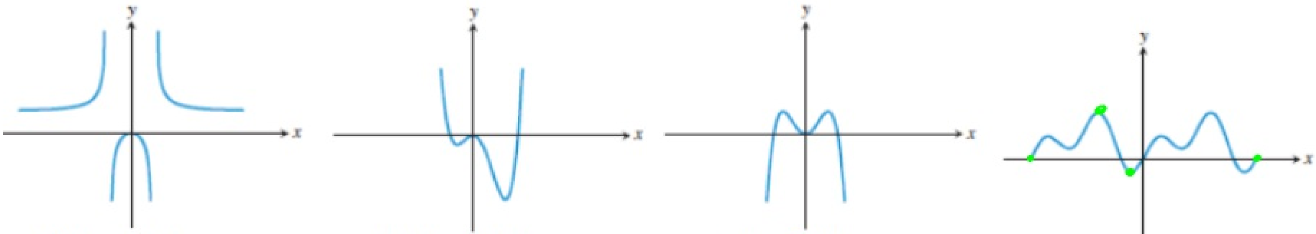


Constant



Decreasing on (−∞, −2]
Constant on [−2, 2]
Increasing on [2, ∞)

Boundedness a way to describe a function

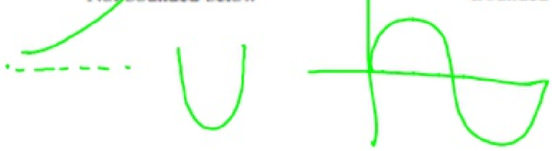


Not bounded above
Not bounded below

Not bounded above
Bounded below

Bounded above
Not bounded below

Bounded

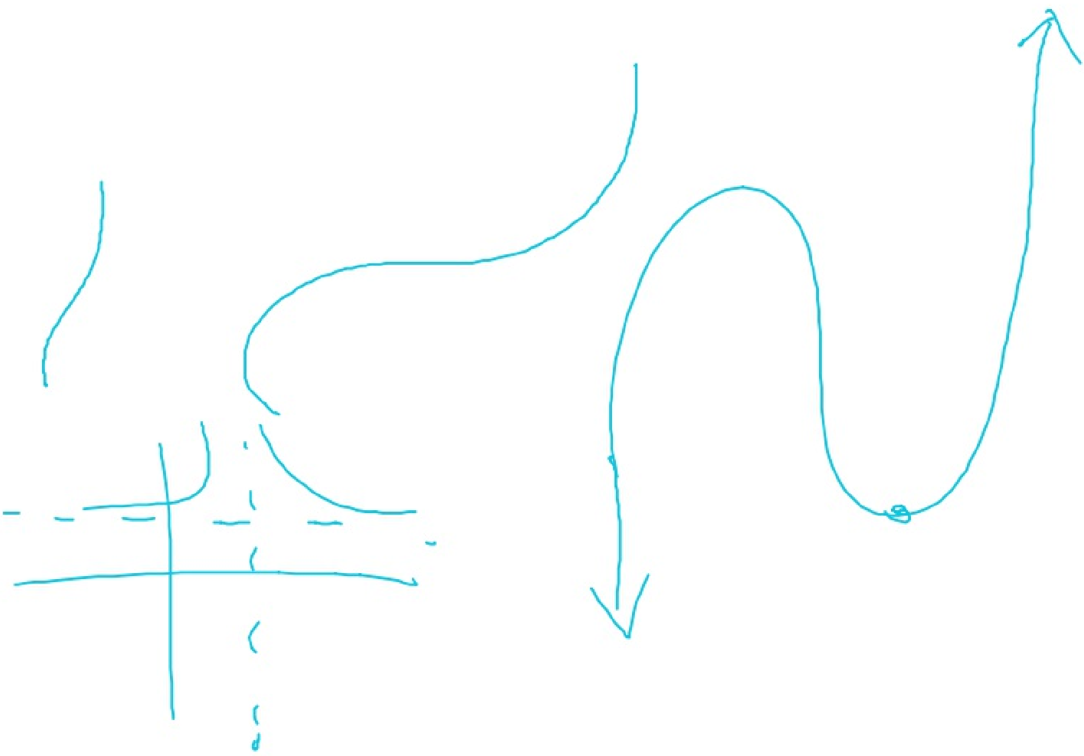


Extrema maximum/minimum

~There are two types of extrema: *absolute* and *relative* or *local*

~Absolute Extrema: in the entire domain, it is ***the*** lowest or highest point

~Relative/Local Extrema: for a particular portion of the domain, it is a low point or a high point.

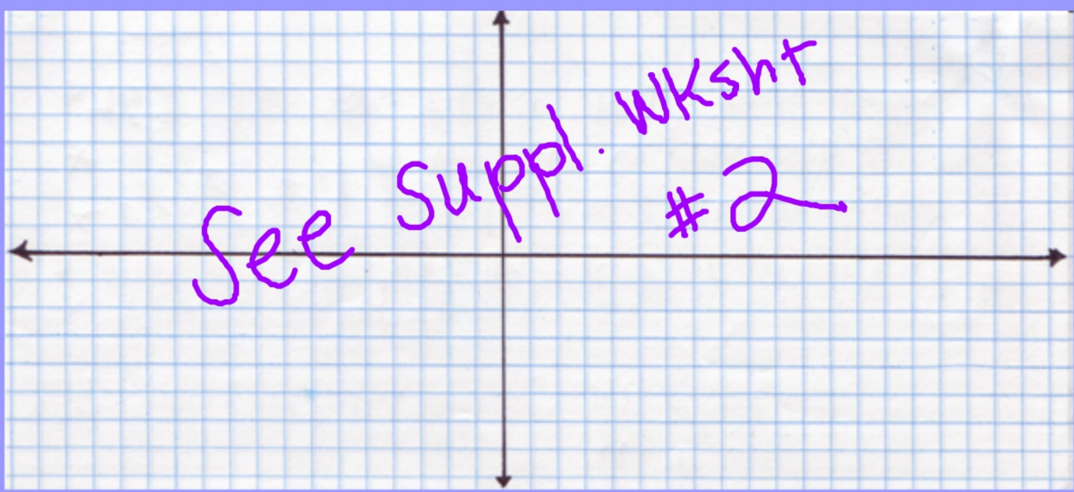


~a *maximum* occurs when the graph changes from increasing to decreasing

~a *minimum* occurs when the graph changes from decreasing to increasing

*** extrema are always written as an ordered pair***

Ex State the intervals for increasing, decreasing and constant. Then state all relative extrema.



Warm-up for 1/28/14

Factor completely. If not factorable, write prime.

1. $x^2 - x - 12$

2. $4x^2 - 9$

3. $5x^2 - 12x + 4$

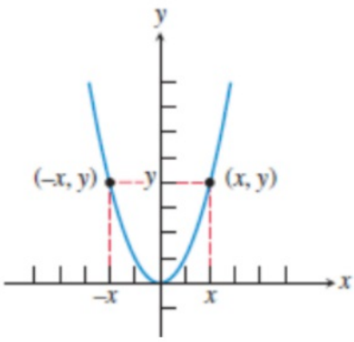
Symmetry

We study two kinds of symmetry: y-axis and orign.

*y-axis symmetry:

algebraically: $f(-x)=f(x)$ *translation: opposite x-values yield the same y-value*

geometrically:

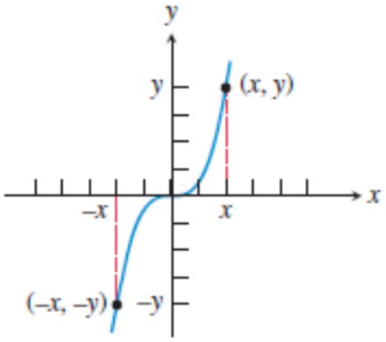


**Functions with y-axis symmetry are called "even functions"*

origin symmetry

Symmetry with respect to the origin
Example: $f(x) = x^3$

Graphically



Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

Ex Determine whether the function has even, odd or neither symmetry.

~to do this...plug in a negative *x* and see what the function looks once it is simplified completely.

*if it is the original function exactly---EVEN

*if it is the exact opposite of the function----ODD

*if it is some combo----NEITHER

A. $f(x) = x^2 - 3$

$f(-x) = (-x)^2 - 3$

$f(-x) = x^2 - 3$

even

B. $g(x) = x^2 - 2x - 2$

$g(-x) = (-x)^2 - 2(-x) - 2$

$g(-x) = x^2 + 2x - 2$

Neither

C. $h(x) = \frac{x^3}{4 - x^2}$

$h(-x) = \frac{(-x)^3}{4 - (-x)^2}$

$h(-x) = \frac{-x^3}{4 - x^2}$

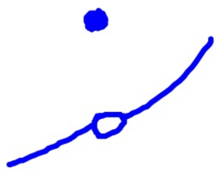
$h(-x) = -\left(\frac{x^3}{4 - x^2}\right)$

odd

$\frac{-2}{3}$ $\frac{2}{-3}$ $-\left(\frac{2}{3}\right)$

Closure: Complete Worksheet 1.2 with a partner.

1. $(-17, -15) \cup (-12, -10) \cup (-7, 3) \cup (8, 12) \cup (15, \infty)$
2. $(-15, -12) \cup (-10, -7) \cup (3, 8)$
3. $(12, 15)$
4. $[-17, 3) \cup (3, \infty)$
5. $[-7, \infty)$
6. $(-12, -6)$, $(-7, -7)$ and $(8, -5)$
7. $(-15, 4)$, $(-10, -3)$
8. None
9. $(-7, -7)$
10. Not continuous / removable @ $x = -1$, infinite @ $x = 3$, jump @ $x = 6$
11. Yes, it is bounded below by $y = -7$
12. 3



$f(-1)$

Ex Given: $(3, -1)$

A. origin sym.

$(-3, 1)$

B. even function

$(-3, -1)$

Lesson Title: 1.3 Twelve Basic Functions & 1.6 Graphical Transformations

Objectives: SWBAT sketch a parabola, line, cubic, square root, absolute value, reciprocal, exponential, natural logarithm, logistics, greatest integer function and constant functions by shifting, reflecting and stretching the mother graph; graph and evaluate piecewise-defined functions; state end behavior using limit notation; discuss continuity.

Agenda:
1. finish 1.2 (symmetry & even/odd/neither)
2. Notes: 1.3
3. Closure: level of understanding (thumbs up/down)

Homework: p.113 (1-12)(14,15,17,19,21,23,26,27,31,33)(35-41odd)(45,51) & p.147(5,9,14,18,21,22,24)(25-28)(44-46)(48,49)(51-54)

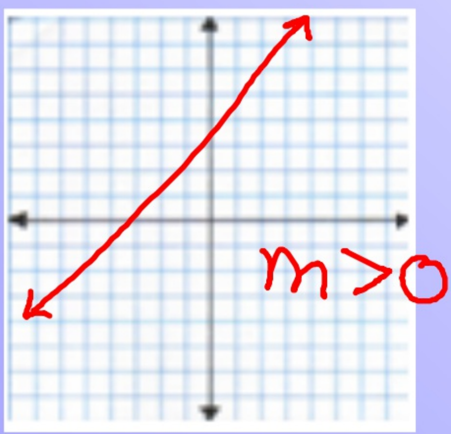
Test/Quiz:

The Library of Functions

The Library of functions provides us with a reference for some of the most used functions in our class. Knowing the graphs of these functions and some key information will allow us to work with any altered form of each.

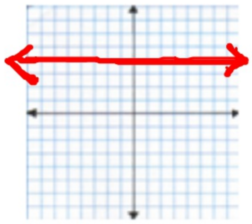
I. Linear Function

- $f(x) = mx + b$, where m and b are elements of the real numbers
- domain: all real numbers if $y \neq 0$
- range: all real numbers if $x \neq 0$
- increasing function if $m > 0$
- decreasing function if $m < 0$
- constant if $m = 0$



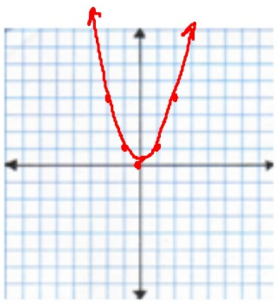
A. Constant function

- $f(x) = b$, where b is an element of the real numbers
- domain: $(-\infty, \infty)$
- range: $\{b\}$
- even function



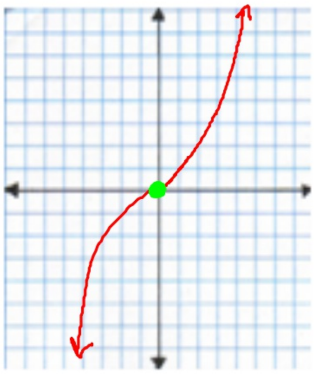
II. Square Function

- $f(x) = x^2$
- domain: $(-\infty, \infty)$
- range: $[0, \infty)$
- even function
- decreasing: $(-\infty, 0)$
- increasing: $(0, \infty)$



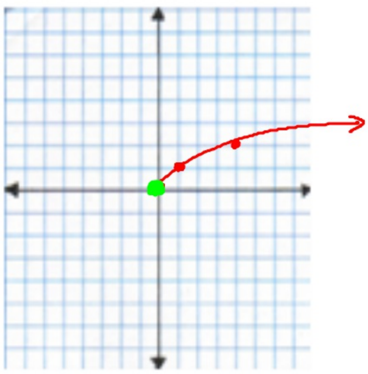
III. Cube Function

- $f(x) = x^3$
- domain: $(-\infty, \infty)$
- range: $(-\infty, \infty)$
- odd function
- increasing: $(-\infty, \infty)$



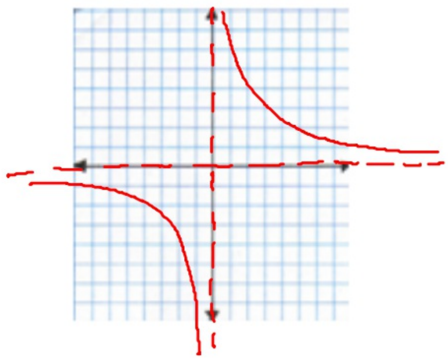
IV. Square Root Function

- $f(x) = \sqrt{x}$
- domain: $[0, \infty)$
- range: $[0, \infty)$
- neither even or odd
- increasing: $[0, \infty)$



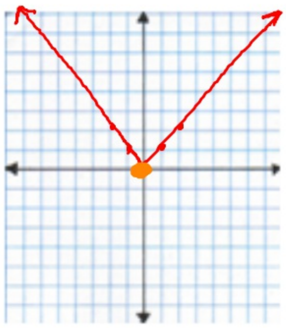
V. Reciprocal Function

- $f(x) = \dfrac{1}{x}$
- domain: $(-\infty, 0) \cup (0, \infty)$
- range: $(-\infty, 0) \cup (0, \infty)$
- odd function
- decreasing: $(-\infty, 0) \cup (0, \infty)$



VI. Absolute Value Function

- $f(x) = |x|$
- domain: $(-\infty, \infty)$
- range: $[0, \infty)$
- even function
- decreasing: $(-\infty, 0)$
- increasing: $(0, \infty)$



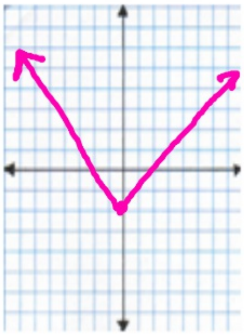
Let's now look at how we can alter these graphs using lifts, shifts and stretches.

I. Vertical Shift

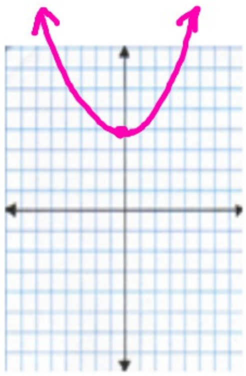
- $f(x)+k$, shifts every point on the "mother graph" up k units
- $f(x)-k$, shifts every point on the "mother graph" down k units

Examples

A. $f(x)=|x|-2$



B. $g(x)=x^2+4$



II. Horizontal Shift

- $f(x-h)$, shifts every point on the "mother graph" to the right h units
- $f(x+h)$, shifts every point on the "mother graph" to the left h units

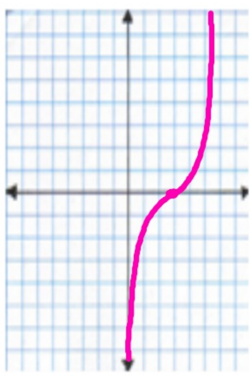
Examples

A. $f(x)=(x-3)^3$

$x-3=0$

$x=3$

R3

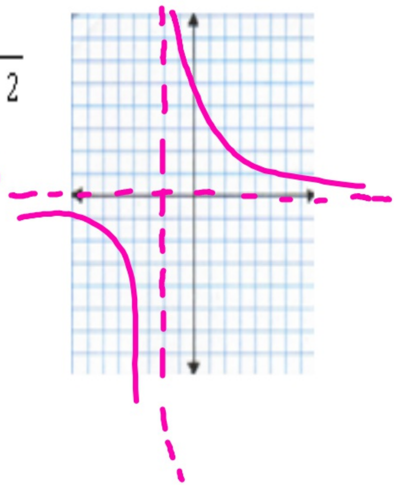


B. $g(x)=\dfrac{1}{x+2}$

$x+2=0$

$x=-2$

L2

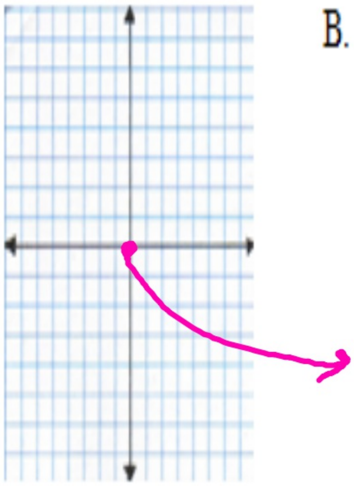


III. X-axis Reflection

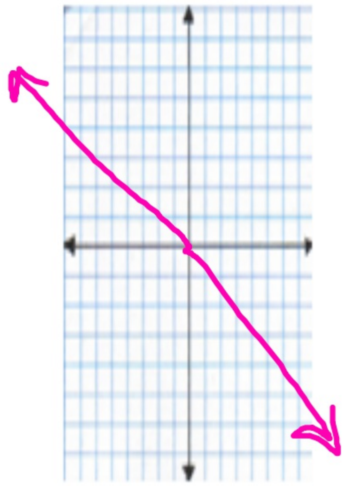
- $-f(x)$, changes the sign on all the y-coordinates of the "mother graph"

Examples

A. $f(x) = -\sqrt{x}$



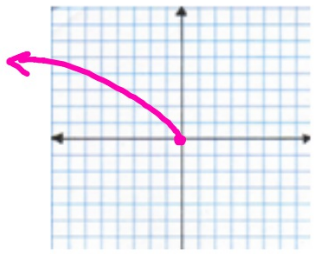
B. $g(x) = -x$



IV. Y-axis Reflection

- $f(-x)$, changes the sign on all the x-coordinates of the "mother graph"

Example: $f(x) = \sqrt{-x}$



V. Stretches

- $af(x)$

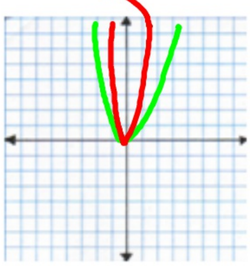
  A. if $a > 1$, then the y-coordinates of the "mother graph" are a times larger; pulls the graph closer to the y-axis.

  B. if $0 < a < 1$, then the y-coordinates of the "mother graph" are a times smaller; pulls the graph closer to the x-axis.

Examples

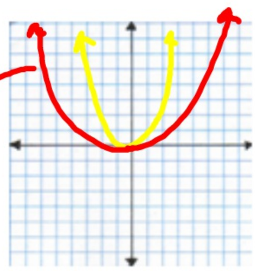
A. $f(x) = 2x^2$

narrow



B. $g(x) = \frac{1}{4}x^2$

wider



All of the above alterations can be put together in any number of combinations. (I find it easier to do the reflections before any other step!)

Examples

A. $f(x) = -(x+5)^2 - 1$

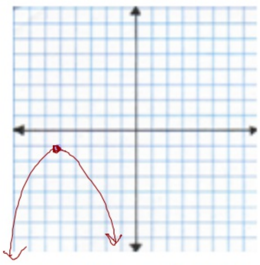
x-refl

$x+5=0$

$x=-5$

L5

D1



B. $g(x) = \sqrt{3-x} + 3$

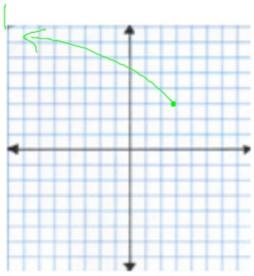
y-refl

$3-x=0$

$x=3$

R3

U3



C. $f(x) = 3(x-2)^3 + 4$

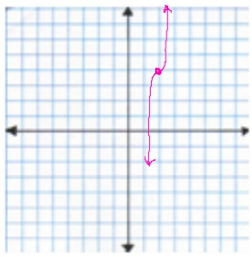
narrow

$x-2=0$

$x=2$

R2

U4



D. $g(x) = -\frac{1}{2}|x+5| - 1$

wider

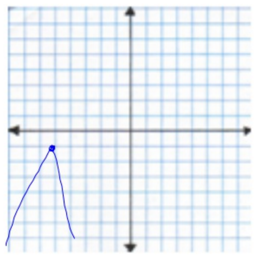
x-refl

D1

$x+5=0$

$x=-5$

L5



Piece Wise Defined Functions

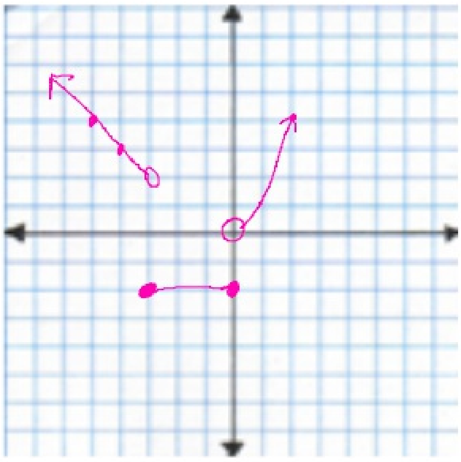
~"frankenstein" functions

~each level of the function represents a new portion of the graph and has its own domain restrictions

~should be able to graph and evaluate

Ex Sketch the graph.

$$f(x)=\begin{cases} x^2, & x>0 \quad \text{I} \\ -2, & -3\le x\le 0 \quad \text{II} \\ -x, & x< -3 \quad \text{III} \end{cases}$$



I.

	x	$y=x^2$
open	0	0
	1	1
	2	4
	kg	

II.

	x	$y=-2$
closed	-3	-2
	-2	-2
	-1	-2
closed	0	-2

III.

	x	$y=-x$
~~closed~~ open	-3	3
	-4	4
	-5	+5
	kg	

<u>Ex</u> Evaluate the function for the given values of *x*.

$$f(x) = \begin{cases} 3 - x^2 , & x > -3 \\ 2x + 1 , & x \le -3 \end{cases}$$

A. *f(-7)*

$2(-7) + 1$

$-14 + 1$

-13

B. *f(-3)*

$2(-3) + 1$

-5

C. *f(0)*

$3 - (0)^2$

$\boxed{3}$