Homework tonight: finish book work from last night and the new worksheet(1 & 3) p 468

#6

$\sin \frac{7\pi}{12}$

$\frac{7\cancel{\pi}}{\cancel{12}_{1}} \cdot \frac{\cancel{180}^{\circ}\,\cancel{30}\,15}{\cancel{\pi}}$

105°

$45^{\circ}+60^{\circ}$

$\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$\boxed{\sin\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos\frac{\pi}{4}}$

$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

7. $\tan\left(\frac{5\pi}{12}\right)$

$$\frac{5\cancel{\pi}}{\cancel{12}} \cdot \frac{\cancel{180^\circ}\,15}{\cancel{\pi}}$$

75°

$45^\circ + 30^\circ$

$$\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$

$$\frac{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}}}$$

$$\frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}} \cdot \frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

12. $\cos 94^\circ \cos 18^\circ + \sin 94^\circ \sin 18^\circ$

$\cos(94^\circ - 18^\circ)$

$\cos(76^\circ)$

P475 #16

$$\cos(6x) = 2\cos^2(3x) - 1$$

$$\cos(2 \cdot \underbrace{3x}_{u}) = 2\cos^2(3x) - 1$$

$$\cos(2u) = 2\cos^2 u - 1$$

23. $\sin\left(x-\frac{\pi}{2}\right) = -\cos x$

$\sin(u-v)$

$\sin x \cos\frac{\pi}{2} - \sin\frac{\pi}{2}\cos x = -\cos x$

$\cancel{\sin x(0)} - 1\cdot\cos x = -\cos x$

$-\cos x = -\cos x$

$$\cos(6x) = 2\cos^2(3x) - 1$$

$$\cos(2 \cdot \underbrace{3x}) = 2\cos^2(3x) - 1$$

$$2\cos^2(3x) - 1 = 2\cos^2(3x) - 1$$

$$\cos(6x) = 2\cos^2(3x) - 1$$

$$\cos(3x + 3x)$$

$$\cos(3x)\cos(3x) - \sin(3x)(\sin(3x)) = 2\cos^2(3x) - 1$$

$$\cos^2(3x) - \sin^2(3x) =$$

$$\cos^2(3x) - \left[1 - \cos^2(3x)\right] =$$

$$\cos^2(3x) - 1 + \cos^2(3x)$$

$$2\cos^2(3x) - 1$$

18. $2\cot(2x) = \cot x - \tan x$

$$2 \cdot \frac{1}{\tan(2x)}$$

$$2 \cdot \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$$

$$\cancel{2} \cdot \frac{1-\tan^2 x}{\cancel{2}\tan x} = \cot x - \tan x$$

$$\frac{1-\tan^2 x}{\tan x}$$

$$\frac{1}{\tan x} - \frac{\tan^2 x}{\tan x} =$$

$$\cot x - \tan x =$$

5. $\cos\left(\frac{\pi}{12}\right)$

$\frac{\cancel{\pi}}{\cancel{12}} \cdot \frac{\cancel{180}^{15}}{\cancel{\pi}}$

15°

$45^\circ - 30^\circ$

$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

40. $\cos^3 x = \left(\frac{1}{2}\cos x\right)\left(1 + \cos(2x)\right)$

$$= \frac{1}{2}\cos x\left(\cancel{1} + \boxed{2\cos^2 x - \cancel{1}}\right)$$

$$= \cancel{\frac{1}{2}}\cos x\left(\cancel{2}\cos^2 x\right)$$

$$= \cos^3 x$$

Ex Given $\sin u = \frac{-5}{13}$ & $\frac{3\pi}{2} < u < 2\pi$

a) find $\sin(2u)$

$2\sin u \cos u$

$2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right)$

$\frac{-120}{169}$

b) find $\cos(2u)$

$1 - 2\sin^2 u$

$1 - 2\left(\frac{-5}{13}\right)^2$

$1 - \frac{2}{1}\left(\frac{25}{169}\right)$

$\frac{169}{169} - \frac{50}{169}$

$\frac{119}{169}$

Ex Express $\sin(3x)$ in terms of $\sin x$.

$\sin(3x)$

$\sin(2x + x)$ $\quad\quad$ $\sin(u+v) = \sin u \cos v + \sin$

$\sin(2x)\cos x + \sin x \cos(2x)$ $\quad\quad$ $\sin 2u = 2\sin u \cos$

$2\sin x \cos x \cos x + \sin x(1 - 2\sin^2 x)$ $\quad\quad$ $\cos 2u = 1 - 2\sin^2$

$2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$3\sin x - 4\sin^3 x$

Half Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

must choose + **or** − based on Quad of $\frac{u}{2}$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

Ex Given $\sin(105°)$, find the exact value using a half $\angle$ formula.

$\sin(105°)$ Quadrant II ; sine has a pos sign

$$\sin\left(\frac{\overset{u}{210°}}{2}\right) = \sqrt{\frac{1-\cos 210°}{2}}$$

$$= \sqrt{\frac{\frac{2}{2} + \left(+\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

ex) $\sin(105^\circ)$ find the exact value using half $\angle$ formulas

$u = 210^\circ$

$\frac{u}{2} = 105^\circ \rightarrow$ Quad II $\rightarrow$ sine has pos. sign

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}}$$

$$\sin\left(\frac{210^\circ}{2}\right) = \sin(105^\circ) = \sqrt{\frac{1-\cos 210^\circ}{2}}$$

$$\sin(105^\circ) = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$\sin(105^\circ) = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}$$

$$\sin(105^\circ) = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$$

$$\sin(105^\circ) = \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}}$$

$$\sin(105^\circ) = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Ex Rewrite the problem w/ cosine to the first power.

A. $\sin^4 x$

$\sin^2 x \cdot \sin^2 x$

$$\left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1-\cos(2x)}{2}\right)$$

power reduce

$$\frac{1-2\cos(2x)+\cos^2(2x)}{4}$$

$$\frac{\frac{\cancel{1}\,2}{2}-\frac{\cancel{2}\,4\cos(2x)}{2}+\left(\frac{1+\cos(4x)}{2}\right)}{4}$$

$$\frac{\frac{2-4\cos(2x)+1+\cos(4x)}{2}}{4}$$

$$\frac{3-4\cos(2x)+\cos(4x)}{2}\cdot\frac{1}{4}$$

$$\frac{3-4\cos(2x)+\cos(4x)}{8}$$

B. $\cos^4 x$

$$\cos^2 x \cdot \cos^2 x$$

$$\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$$

$$\frac{1+2\cos(2x)+\boxed{\cos^2(2x)}}{4}$$

$$\frac{1+2\cos(2x)+\left(\frac{1+\cos 4x}{2}\right)}{4}$$

$$\frac{2+4\cos(2x)+1+\cos(4x)}{2}\cdot\frac{1}{4}$$

$$\frac{3+4\cos(2x)+\cos(4x)}{8}$$