

⑬ $y = 1 + 2\csc(\pi x - 3)$

$a = 2$ $b = \pi$ $c = -3$ $d = 1$

~~amp = 2~~
~~none~~

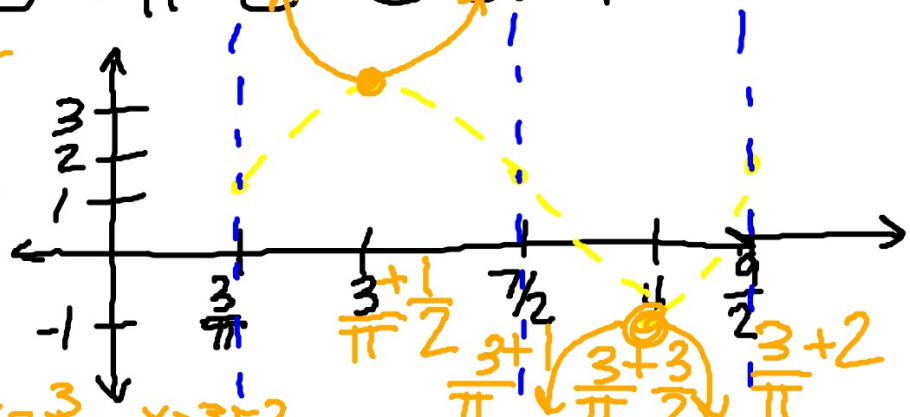
period = 2

PS = $\frac{3}{\pi}$

VS = 1

Asym = $x = \frac{3}{\pi}$ $x = \frac{3}{\pi} + 2$

Key Points: ~~$(\frac{3}{\pi}, 1)$~~ ~~$(\frac{3+\frac{1}{2}}{\pi}, 3)$~~ ~~$(\frac{7}{2}, 1)$~~ ~~$(4, -1)$~~
 ~~$(\frac{9}{2}, 1)$~~



$$\textcircled{3} \quad y = \frac{1}{3} \sin(\pi x + 2) - 4$$

$$a = \frac{1}{3} \quad \text{ampl. } \frac{1}{3}$$

$$b = \pi \quad \text{period} = 2$$

$$c = 2 \quad \text{P.S. } -2$$

$$d = -4 \quad \frac{2}{4} = \frac{1}{2} \quad -\frac{2}{\pi} + \frac{1}{2}$$

$$\textcircled{\frac{P}{4}}$$

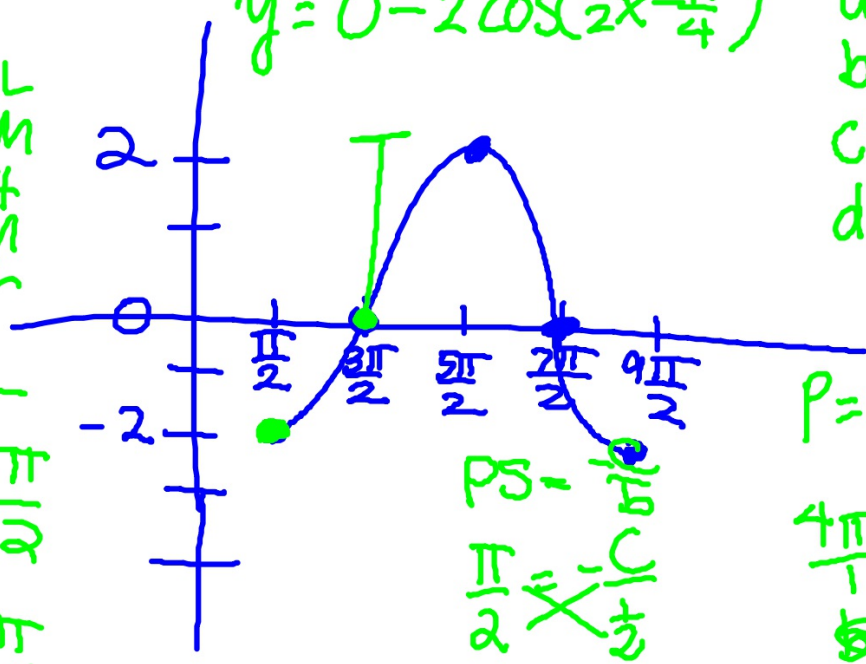
M	H	M
$\left(\frac{-2}{\pi}, -4 \right)$	$\left(-\frac{2}{\pi} + \frac{1}{2}, -4 + \frac{1}{3} \right)$	$\left(\frac{-2}{\pi} + 1, -4 \right)$
L	M	
$\left(-\frac{2}{\pi} + \frac{3}{2}, -4 + \frac{1}{3} \right)$	$\left(-\frac{2}{\pi} + 2, -4 \right)$	

15.

$$y = 0 - 2 \cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$$

$$\begin{aligned} a &= 2 \\ b &= \frac{1}{2} \\ c &= -\frac{\pi}{4} \\ d &= 0 \end{aligned}$$

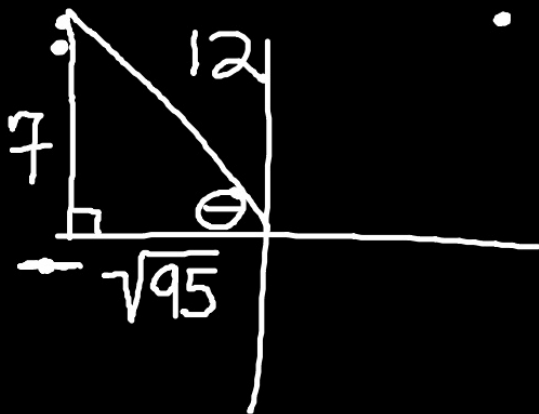
$\frac{0}{\frac{\pi}{2}}$
 $\frac{1}{\pi}$
 $\frac{2}{\frac{3\pi}{2}}$
 $\frac{3}{2\pi}$
 $\frac{4}{\frac{5\pi}{2}}$
 $\frac{5}{3\pi}$
 $\frac{6}{2\pi}$
 $\frac{7}{\frac{9\pi}{2}}$
 $\frac{8}{\pi}$
 $\frac{9}{\frac{3\pi}{2}}$
 $\frac{10}{2\pi}$
 $\frac{11}{\frac{5\pi}{2}}$
 $\frac{12}{\pi}$



$$\begin{aligned} \text{PS} &= \frac{\pi}{2} \\ &= -\frac{c}{b} \\ &= -\frac{-\frac{\pi}{4}}{\frac{1}{2}} \\ &= -2c \\ c &= -\frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} P &= \frac{2\pi}{b} \\ \frac{4\pi}{1} &= \frac{2\pi}{\frac{1}{2}} \\ 4\pi \cdot \frac{1}{2} &= 2\pi \\ b &= \frac{1}{2} \end{aligned}$$

D. $\sin \theta = \frac{7}{12}$ & $\cos \theta < 0$



$$12^2 = 7^2 + b^2$$

$$144 = 49 + b^2$$

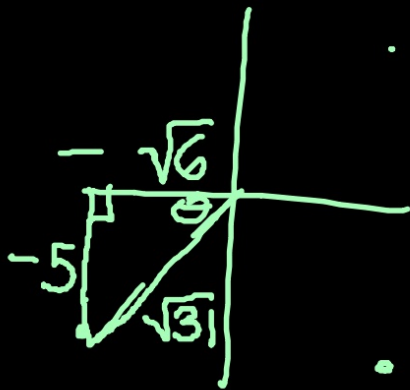
$$95 = b^2$$

$$b = \sqrt{95} \approx 9.7$$

$\sin \theta = \frac{7}{12}$	$\csc \theta = \frac{12}{7}$	
$\cos \theta = \frac{-\sqrt{95}}{12}$	$\sec \theta = \frac{-12}{\sqrt{95}}$	→ or $-\frac{12\sqrt{95}}{95}$
$\tan \theta = \frac{-7}{\sqrt{95}}$	$\cot \theta = \frac{-\sqrt{95}}{7}$	

or $-\frac{7\sqrt{95}}{95}$

E. $\csc \theta = -\frac{\sqrt{31}}{5}$ & $\tan \theta > 0$



$$(\sqrt{31})^2 = (-5)^2 + x^2$$

$$31 = 25 + x^2$$

$$x = \sqrt{6}$$

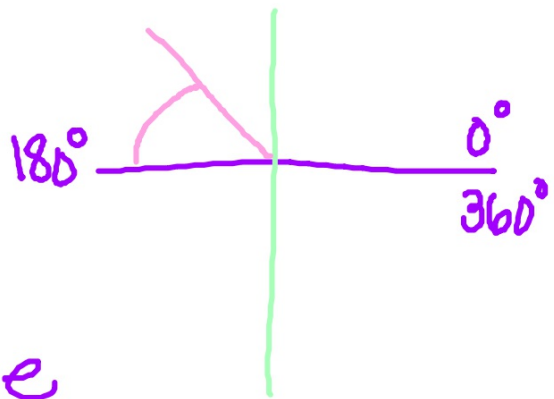
$$\sin \theta = \frac{-5}{\sqrt{31}}$$

$$\cos \theta = \frac{-\sqrt{6}}{\sqrt{31}}$$

$$\tan \theta = \frac{+5}{+\sqrt{6}}$$

Reference Angles

- always +
- always acute
- describe distance between the \angle & the closest x-axis value



Ex 125°
what's its
ref \angle ?

$$180^\circ - 125^\circ$$

55°

B. 301°

$360^\circ - 301^\circ$

59°

C. $\frac{3\pi}{4}$

I am
 135°
 $\therefore 45^\circ$

$\frac{\pi}{4}$

D. $\frac{-11\pi}{2}$

$+2\pi$

$\frac{-7\pi}{2}$

$+2\pi$

$\frac{-3\pi}{2}$

$+2\pi$

$\frac{\pi}{2}$