

**Objective:** Find the partial fraction decomposition for rational expressions (what two or more fractions add to give you this rational expression)

## 7.4 Partial Fraction Decomposition

### Steps

1. Factor the denominator completely
2. Set = to a sum of rational fractions with a variable in the numerator for each factor.
3. Multiply by common denominator.
4. Distribute and collect the terms with x
5. Set up system and solve for A and B

Ex1.  $\frac{x+7}{x^2-x-6}$

$$\frac{(x+7)\cancel{(x-3)(x+2)}}{\cancel{(x-3)(x+2)}} = \frac{A\cancel{(x-3)(x+2)}}{\cancel{x-3}} + \frac{B\cancel{(x-3)(x+2)}}{\cancel{x+2}}$$

$$x+7 = A(x+2) + B(x-3)$$

$$x+7 = Ax + 2A + Bx - 3B$$

$$\begin{aligned} x &= Ax + Bx \\ 1 &= A + B \end{aligned}$$

$$\begin{cases} 7 = 2A - 3B \\ 1 = A + B - 2 \end{cases} \Rightarrow \begin{aligned} 7 &= 2A - 3B \\ -2 &= 2A - 2B \end{aligned}$$

$$5 = -5B$$

$$B = -1$$

$$\begin{aligned} 1 &= A - 1 \\ A &= 2 \end{aligned}$$

$$\frac{x+7}{x^2-x-6} = \frac{2}{x-3} - \frac{1}{x+2}$$

$$\frac{2(x+2)}{(x-3)(x+2)} - \frac{1(x-3)}{(x+2)(x-3)}$$

$$\frac{x+7}{x^2-x-6}$$

Ex2.  $\frac{9x-2}{x^2+x-6}$

$$\frac{(9x-2)(x+3)(x-2)}{(x+3)(x-2)} = \frac{A(x+3)(x-2)}{x+3} + \frac{B(x+3)(x-2)}{x-2}$$

$$9x-2 = A(x-2) + B(x+3)$$

$$9x-2 = Ax - 2A + Bx + 3B$$

$$9x = Ax + Bx$$

$$-2 = -2A + 3B$$

$$\begin{cases} 9 = A + B \\ -2 = -2A + 3B \end{cases} \Rightarrow \begin{array}{l} 18 = 2A + 2B \\ -2 = -2A + 3B \\ \hline 16 = 5B \end{array}$$

$$9 = A + B$$

$$9 = A + \frac{16}{5}$$

$$\frac{9}{5} - \frac{16}{5} = A$$

$$\frac{29}{5} = A$$

$$16 = 5B$$

$$B = \frac{16}{5}$$

$$\frac{9x-2}{x^2+x-6} = \frac{29}{5(x+3)} + \frac{16}{5(x-2)}$$

Ex3.  $\frac{x^2+12x+3}{x^3-4x}$

$$\frac{x^2+(2x+3)}{x(x^2-4)}$$

$$\frac{(x^2+12x+3)}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

$$x^2+12x+3 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$$

$$x^2+12x+3 = A(x^2-4) + Bx^2+2Bx + Cx^2 - 2Cx$$

$$x^2+12x+3 = Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$$

$x^2 = Ax^2 + Bx^2 + Cx^2$	$12x = 2Bx - 2Cx$	$3 = -4A$
$1 = A + B + C$	$12 = 2B - 2C$	$A = -\frac{3}{4}$
$1 = -\frac{3}{4} + B + C$		



Ex4.  $\frac{5x^2+24x+2}{x^3+4x^2+4x}$

$$\frac{5x^2+24x+2}{x(x^2+4x+4)}$$

$$\frac{5x^2+24x+2}{x(x+2)(x+2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$5x^2+24x+2 = A(x+2)(x+2) + Bx(x+2) + Cx$$

$$5x^2+24x+2 = A(x^2+4x+4) + Bx^2+2Bx + Cx$$

$$5x^2+24x+2 = Ax^2+4Ax+4A+Bx^2+2Bx+Cx$$

$$5x^2 = Ax^2 + Bx^2 \quad \left\{ \begin{array}{l} 24x = 4Ax + 2Bx + Cx \\ 24 = 4A + 2B + C \end{array} \right. \quad \left\{ \begin{array}{l} 2 = 4A \\ A = \frac{1}{2} \end{array} \right.$$

$$5 = A + B$$

$$5 = \frac{1}{2} + B$$

$$5 - \frac{1}{2} = B$$

$$\frac{9}{2} = B$$

$$24 = 2 + 9 + C$$

$$C = 13$$

$$\frac{5x^2+24x+2}{x^3+4x^2+4x} = \frac{1}{2(x)} + \frac{9}{2(x+2)} + \frac{13}{(x+2)^2}$$

**Homework**  
**p.614 #5, 6, 21, 22, 23**