Objective: Find the partial fraction decomposition for rational expressions (what two or more fractions add to give you this rational expression)

7.4 Partial Fraction Decomposition

Steps

1. Factor the denominator completely
2. Set = to a sum of rational fractions with a variable in the numerator for each factor.
3. Multiply by common denominator.
4. Distribute and collect the terms with x
5. Set up system and solve for A and B

Ex1. $\dfrac{x+7}{x^2-x-6}$

$$\frac{(x+7)(x-3)(x+2)}{(x-3)(x+2)} = \frac{A(x-3)(x+2)}{x-3} + \frac{B(x-3)(x+2)}{x+2}$$

$$x+7 = A(x+2) + B(x-3)$$

$$x+7 = Ax+2A+Bx-3B$$

$$\frac{x = Ax+Bx}{x}$$

$$1 = A+B$$

$$\begin{cases} 7 = 2A-3B \\ (1 = A+B)\cdot(-2) \end{cases} \Rightarrow \begin{array}{l} 7 = 2A-3B \\ -2 = -2A-2B \\ \hline 5 = -5B \end{array}$$

$$B = -1$$

$$1 = A-1$$

$$A = 2$$

$$\frac{x+7}{x^2-x-6} = \frac{2}{x-3} - \frac{1}{x+2}$$

$$\frac{2(x+2)}{(x-3)(x+2)} - \frac{1(x-3)}{(x+2)(x-3)}$$

$$\frac{1x+7}{}$$

Ex2. $\dfrac{9x-2}{x^2+x-6}$

$$\frac{(9x-2)(x+3)(x-2)}{(x+3)(x-2)} = \frac{A(x+3)(x-2)}{x+3} + \frac{B(x+3)(x-2)}{x-2}$$

$$9x-2 = A(x-2)+B(x+3)$$

$$9x-2 = Ax-2A+Bx+3B$$

$$9x = Ax+Bx$$
$$-2 = -2A+3B$$

$$\begin{cases} (9 = A+B)2 \Rightarrow 18 = 2A+2B \\ -2 = -2A+3B \end{cases}$$
$$-2-2A+3B$$
$$16 = 5B$$
$$B = \frac{16}{5}$$

$$9 = A+B$$
$$9 = A+\frac{16}{5}$$
$$\frac{45}{5} - \frac{16}{5} = A$$
$$\frac{29}{5} = A$$

$$\frac{9x-2}{x^2+x-6} = \frac{29}{5(x+3)} + \frac{16}{5(x-2)}$$

Ex3. $\dfrac{x^2+12x+3}{x^3-4x} = \dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$

$x(x^2-4)$

$x(x-2)(x+2)$

$x^2+12x+3 = A(x+2)(x-2)+Bx(x+2)+Cx(x-2)$

$x^2+12x+3 = Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$

$1 = A+B+C$ | $12 = 2B-2C$ | $3 = -4A$

$A = -\frac{3}{4}$

$1 = -\frac{3}{4}+B+C$

$\frac{7}{4} = B+C$

$\begin{cases} \frac{7}{4} = B+C \\ 12 = 2B-2C \end{cases}$ / 2

$\begin{cases} \frac{7}{4} = B+C \\ 6 = B-C \end{cases}$

$\frac{31}{4} = 2B$

$\frac{31}{4}\cdot\frac{1}{2} = B$

$B = \frac{31}{8}$

$6 = B-C$

$6 = \frac{31}{8}-C$

$\frac{48}{8}-\frac{31}{8} = -C$

$-\frac{17}{8} = C$

$\dfrac{x^2+12x+3}{x^3-4x} = \dfrac{-3}{4x} + \dfrac{31}{8(x-2)} - \dfrac{17}{8(x+2)}$

$\dfrac{-\frac{3}{4}}{x}$

$-\frac{3}{4}\cdot\frac{1}{x}$

$\dfrac{-3}{4x}$

Ex4. $\dfrac{5x^2+24x+2}{x^3+4x^2+4x}$

Repeating Factors

$$\frac{5x^2+24x+2}{x(x+2)(x+2)} = \frac{A}{x} + \frac{B}{(x+2)^1} + \frac{C}{(x+2)^2}$$

$$5x^2+24x+2 = A(x+2)(x+2) + Bx(x+2) + Cx$$

$$5x^2+24x+2 = A(x^2+4x+4) + Bx^2+2Bx + Cx$$

$$5x^2+24x+2 = Ax^2+4Ax+4A+Bx^2+2Bx+Cx$$

$5x^2 = Ax^2+Bx^2$ | $24x = 4Ax+2Bx+Cx$ | $2 = 4A$

$5 = A+B$ | $24 = 4A+2B+C$ | $A = \frac{1}{2}$

$5 = \frac{1}{2}+B$ | $24 = 2+2B+C$

$B = \frac{9}{2}$ | $22 = 2B+C$

$$22 = 2\left(\frac{9}{2}\right)+C$$

$$22 = 9+C$$

$$C = 13$$

$$\frac{5x^2+24x+2}{x^3+4x^2+4x} = \frac{1}{2x} + \frac{9}{2(x+2)} + \frac{13}{(x+2)^2}$$

Homework

p.614 #5, 6, 21, 22, 23

23. $\frac{2x^2+5}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)^1} + \frac{Cx+D}{(x^2+1)^2}$

$(x^2+1)(x^2+1)$

$2x^2+5 = (Ax+B)(x^2+1) + Cx+D$

$2x^2+5 = Ax^3+Ax+Bx^2+B+Cx+D$

$0x^3 = Ax^3$ | $2x^2 = Bx^2$ | $0x = Ax+Cx$ | $5 = B+D$

$0 = A$ | $2 = B$ | $0 = A+C$ | $5 = 2+D$

$0 = 0+C$ | $D = 3$

$C = 0$

$0x+2$ $\qquad$ $0x+3$

$$\frac{2x^2+5}{(x^2+1)^2} = \frac{2}{x^2+1} + \frac{3}{(x^2+1)^2}$$

$$\frac{\text{wwww}}{x^2+4x+4} = \frac{A}{(x+2)^1} + \frac{B}{(x+2)^2}$$

$$\frac{(x+2)(x+2)}{(x+2)}$$

$$\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

(23) $\dfrac{2x^2+5}{(x^2+1)^2} = \dfrac{Ax+B}{(x^2+1)^1} + \dfrac{Cx+D}{(x^2+1)^2}$

$(x^2+1)(x^2+1)$

$2x^2+5 = (Ax+B)(x^2+1) + Cx+D$

$0x^3 + 2x^2 + 0x + 5 = Ax^3 + Ax + Bx^2 + B + Cx + D$

$0x^3 = Ax^3$ | $2x^2 = Bx^2$ | $0x = Ax + Cx$ | $5 = B + D$

$0 = A$ | $2 = B$ | $0 = A + C$ | $5 = 2 + D$

$0 = 0 + C$ | $D = 3$

$C = 0$

$$\frac{2x^2+5}{(x^2+1)^2} = \frac{2}{(x^2+1)} + \frac{3}{(x^2+1)^2}$$