Law of Sines, Law of Cosines and Area of Oblique Triangles

*Used for oblique triangles (non-right triangles)
*Used to find a side or an angle
*Use Law of Sines only when a "side-angle" combo is present
    *if L of S, watch for SSA. (due to hinge)
        *can have no solutions, 1 solution or 2 solutions

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

*Use Law of Cosines when Law of Sines isn't applicable.
    *always find the largest angle first if SSS

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

*Find the area of a triangle without a height:

$$\triangle \text{ Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

s= $(a + b + c)/2.$

or

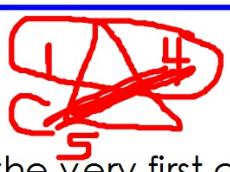
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Things to Think About

*If a side-angle combo then use law of sines, otherwise use law of cosines.

*If using law of sines, watch for SSA...

*if SSA then you have 3 cases:

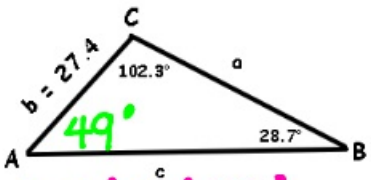


no solution:
You will know this because the very first angle that you find will give you a Domain Error on your calculator

one/two solution(s):
You find the first angle. Then you immediately find the supplement of that angle. Add it to the given angle. If the sum is less than 180, there will be two solutionss. If the Sum is greater than or equal to 180, there is just one solution.

*If given all sides, be sure to find the largest angle first. Otherwise, you may find that some geometry rules are violated (across from the largest angle is the largest side, etc...)

For the triangle, $C = 102.3°$, $B = 28.7°$, and $b = 27.4$ feet. Find the remaining angle and sides.



A	B	C	A	B	C
49°	28.7°	102.3°			
a	b	c	a	b	c
43.1'	27.4'	55.7'			

$A = 180° - 102.3° - 28.7°$

$A = 49°$

$$\frac{27.4}{\sin 28.7°} = \frac{c}{\sin 102.3°}$$

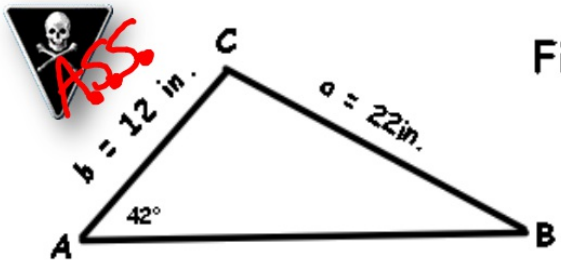
$$c = \frac{27.4 \sin 102.3°}{\sin 28.7°}$$

$c = 55.7$ feet

$$\frac{a}{\sin 49°} = \frac{27.4}{\sin 28.7°}$$

$$a = \frac{27.4 \sin 49°}{\sin 28.7°}$$

$a = 43.1$ ft



Find the remaining side and angles.

$180° - 21.4°$

A		C	A	B	C
42°	21.4°	116.6°	42°	158.6°	
a	b	c	a	b	c
22	12	29.4	22	12	

$$\frac{22}{\sin 42°} = \frac{12}{\sin B}$$

$$\sin B = \frac{12 \sin 42°}{22}$$

$$B = \sin^{-1}\left(\frac{12 \sin 42°}{22}\right)$$

$$B = 21.4°$$

$$C = 180° - 42° - 21.4°$$

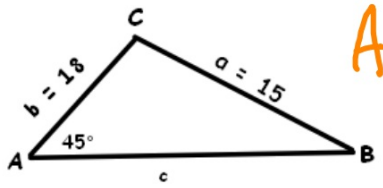
$$C = 116.6°$$

$$\frac{c}{\sin 116.6°} = \frac{22}{\sin 42°}$$

$$c = \frac{22 \sin 116.6°}{\sin 42°}$$

$$c = 29.4 \text{ in}$$

Ex Solve the triangle.



A.S.S.

A	B	C	A	B	C
45	58.1°	76.9°	45	121.9°	13.1°
a	b	c	a	b	c
15	18	20.7	15	18	4.8

A+B< Yes ∴ 2Δ

$$\frac{15}{\sin 45^\circ} = \frac{18}{\sin B}$$

$$B = \sin^{-1}\left(\frac{18\sin 45^\circ}{15}\right)$$

$B = 58.1^\circ$

$C = 180 - 45 - 58.1$

$$\frac{15}{\sin 45^\circ} = \frac{c}{\sin 76.9^\circ}$$

$$c = \frac{15\sin 76.9^\circ}{\sin 45^\circ}$$

$C = 180 - 45 - 121.9$

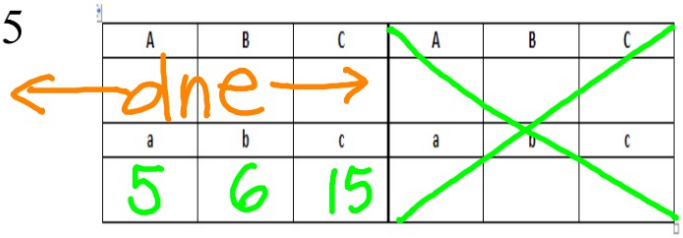
$C = 13.1^\circ$

$$\frac{15}{\sin 45^\circ} = \frac{c}{\sin 13.1^\circ}$$

$c = 4.8$

Ex Find the solution(s) for the triangle with the given pieces:

$a = 5 \quad b = 6 \quad c = 15$



$c^2 = a^2 + b^2 - 2ab\cos C$

$225 = 25 + 36 - 60\cos C$

$225 = 61 - 60\cos C$

$-61 \quad -61$

$\frac{164}{-60} = \frac{-60\cos C}{-60}$

$\cos C = \frac{-164}{60}$

$C = \cos^{-1}\left(\frac{-164}{60}\right)$

domain error