Warm up

1.Find a rectangular equation for each of the following polar equations.

a. $r=6\cos\theta+2\sin\theta$ b. $\theta=\pi/3$

2. Finda set of parametric equations for $y=2x+5$ for the given parameters:

a. $t=\sqrt{x}$ b. $t=x^3$ c. $t=\log x$

3. Find the rectangular equation by eliminating the parameter:

a. $x=t+1$ $y=t^2+3$ b. $x=2^t$ $y=t^2$

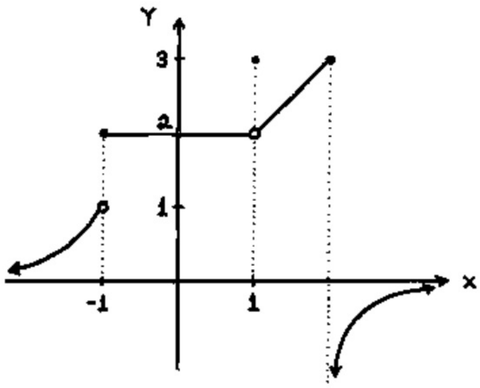
4. Graph the equation $y=2\sin 4\theta$. Give the domain, range and max and min values.

Objective: Define and find a simple limit

If f(x) becomes infinitely close to a value "L" as x approaches c from either side, the "limit of f(x) as x approaches c is L".

What is happening:

- **Y-value as x gets close to c from the right**
- **Y value as x gets close to c from the left**
- **If these values match, this is the limit.**



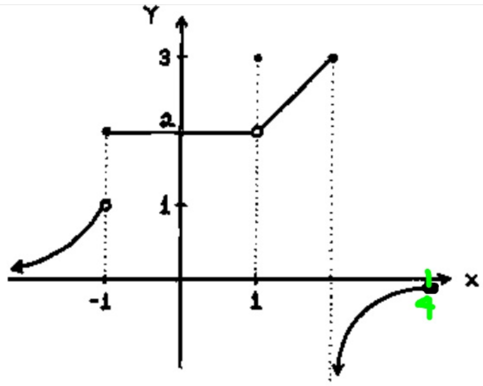
The Limit is always a number or it does not exist (DNE).

Ways for a limit to be DNE.

1) lim left ≠ lim right

*around a VA

*jump discontinuity

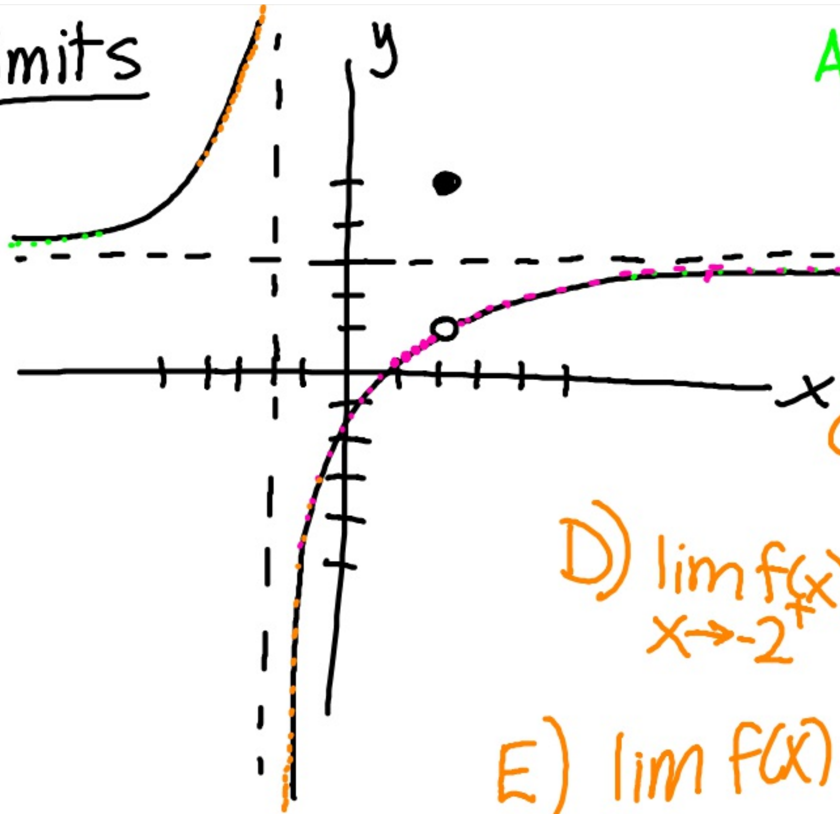


2) No graph on one side of that x value

3) VA at the x value

4) graph oscillates at that x value $y = \sin\left(\frac{1}{x}\right) \Rightarrow x = 0$

Limits



A) $\lim_{x \to \infty} f(x) = 3$

B) $\lim_{x \to -\infty} f(x) = 3$

C) $\lim_{x \to -2^-} f(x) = \infty$

D) $\lim_{x \to -2^+} f(x) = -\infty$

E) $\lim_{x \to -2} f(x) = \text{dne}$

F) $\lim_{x \to 2^-} f(x) = 1$

G) $\lim_{x \to 2^+} f(x) = 1$

H) $\lim_{x \to 2} f(x) = 1$

I) $f(2) = 5$

How to find a limit:

1. Estimating by tables
2. Direct substitution
3. Factor and cancel
4. Rationalizing (conjugates)
5. Common denominators

Estimating by tables

$$\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = 1+1+1$$

1. $f(x) = \dfrac{x^3 - 1^3}{x - 1}$ Find $\lim_{x\to 1} f(x) = 3$

1. type f(x) into y=
2. TABLE SET: start at value of limit change by .000001
3. TABLE- look at, above and below to obtain answer

2. $f(x) = \dfrac{|x|}{x}$ Find $\lim_{x\to 0} f(x) =$ dne

A) $\lim_{x\to 0^-} f(x) = -1$

B) $\lim_{x\to 0^+} f(x) = 1$

If tables aren't allowed...like in "real" math

Direct Substitution

as long as no ÷ by zero occurs

3. Find $\lim\limits_{x \to 2} \frac{x^2 - 6x + 8}{x - 4} = \frac{(2)^2 - 6(2) + 8}{2 - 4}$

$= \frac{4 - 12 + 8}{-2}$

$= 0$

4. Find $\lim\limits_{x \to -3} \frac{x^2 + 4}{x^2 - 1} = \frac{13}{8}$

5. Find $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 2}{x + 3} = \frac{\sqrt{0+1} - 2}{0 + 3} = \frac{1 - 2}{3} = \frac{-1}{3}$

Find $\lim\limits_{x \to -4} \frac{\sqrt{x+4} - 2}{x - 1} = \frac{-2}{-5} > 2$

7. Find $\lim_{x \to 3} \dfrac{\frac{1}{x+3} - \frac{1}{3}}{x+5}$

8. Find $\lim_{x \to 6} \dfrac{\frac{1}{x+1} - \frac{1}{5}}{x-4}$

But what about this?!?!

1. Find $\lim_{x \to 2} \dfrac{x^2 - 6x + 8}{x^2 - 4}$

Factoring

1. Find $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x^2 - 4} = \frac{\quad}{4-4} = \frac{\quad}{0}$

$$\lim_{x \to 2} \frac{(x-4)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} \quad \lim_{x \to 2} \frac{x-4}{x+2} = \frac{2-4}{2+2} = -\frac{1}{2}$$

2. Find $\lim_{x \to -4} \frac{x+4}{x^2 - 16} = \frac{\quad}{(-4)^2 - 16} = \frac{\quad}{0}$

$$\lim_{x \to -4} \frac{\cancel{x+4}}{(x-4)\cancel{(x+4)}}$$

$$\lim_{x \to -4} \frac{1}{x-4} = \frac{1}{-4-4} = -\frac{1}{8}$$

$(a-b)(a+b)$

Rationalizing (think conjugate!)

3. Find $\lim_{x \to 0} \frac{(\sqrt{x+1}-1)}{x} \frac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)}$

$\lim_{x \to 0} \frac{(x+\cancel{1}-\cancel{1})}{x(\sqrt{x+1}+1)}$

$\lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)}$

$\lim_{x \to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

4. Find $\lim_{x \to 0} \frac{(\sqrt{x+4}-2)}{x} \frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)}$

$\lim_{x \to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)}$

$\lim_{x \to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

Simplifying complex fractions

Find $\lim_{x \to 0} \dfrac{\frac{3 \cdot 1}{3(x+3)} - \frac{1(x+3)}{3(x+3)}}{x}$

$$\lim_{x \to 0} \frac{\frac{\cancel{3} - x - \cancel{3}}{3(x+3)}}{x}$$

$$\lim_{x \to 0} \frac{-\cancel{x}}{3(x+3)} \cdot \frac{1}{\cancel{x}} = \frac{-1}{3(0+3)} = \frac{-1}{9}$$

Find $\lim_{x \to 4} \dfrac{\frac{5(1)}{5(x+1)} - \frac{1(x+1)}{5(x+1)}}{x-4}$

$$\lim_{x \to 4} \frac{\overset{-x+4}{5 - x - 1}}{5(x+1)} \cdot \frac{1}{x-4}$$

$$\lim_{x \to 4} \frac{-1(\cancel{x-4})}{5(x+1)} \cdot \frac{1}{\cancel{x-4}} = \frac{-1}{25}$$

Wrap-up

Homework p.822 #6-18 even

WARM UP

1. $\lim_{x\to 3} 5x - 4$
2. $\lim_{x\to 5} \frac{\sqrt{x-1}-2}{x-5}$
3. $\lim_{x\to 0} \frac{\sqrt{x-1}-2}{x-5}$
4. $\lim_{x\to -2} \frac{x+2}{x^2-4}$
5. $\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x}$

6. $(-6)^{2/3} \approx 3.30$

8. $\lim_{x \to \pi} \ln\left(\sin \frac{x}{2}\right) = \ln(1) = 0 = 0$

10. $\frac{a^2 - 1}{a^2 + 1}$ (Since $a^2 + 1 > 0$ for all a, we don't have to worry about division by zero.)

12. **(a)** division by zero

(b) $\lim_{x \to 3} \frac{(x+3)(x-3)}{(x+5)(x-3)} = \lim_{x \to 3} \frac{x+3}{x+5} = \frac{6}{8} = \frac{3}{4}$

14. **(a)** division by zero

(b) $\lim_{x \to 2} \frac{(x-2)(x^2+1)}{x-2} = \lim_{x \to 2} (x^2 + 1) = 5$

16. **(a)** division by zero

(b) $\lim_{x \to -2} \frac{|(x + 2)(x - 2)|}{x + 2}$. Check left- and right-hand limits.

Right: $\lim_{x \to -2^+} \frac{(-1)(x + 2)(x - 2)}{x + 2} = \lim_{x \to -2^+} (-x + 2) = 4$

Left: $\lim_{x \to -2^-} \frac{(x + 2)(x - 2)}{x + 2} = \lim_{x \to -2^-} (x - 2) = -4$

Since $4 \neq -4$, the limit does not exist.

18. **(a)** division by zero

(b) The limit does not exist.

Properties of limits

1. Sum rule $\lim_{x\to c} f(x) + \lim_{x\to c} g(x) = \lim_{x\to c}\left(f(x)+g(x)\right)$

2. Difference rule $\lim_{x\to c}\left(f(x)-g(x)\right) = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$

3. Product rule $\lim_{x\to c}\left(f(x)g(x)\right) = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)$

4. Constant Multiple Rule $\lim_{x\to c} kf(x) = k\left(\lim_{x\to c} f(x)\right)$

5. Quotient rule $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right), g(x) \neq 0$ <u>or</u>

$$\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)};\ g(x) \neq 0$$

$(x-8)^0$

6. Power rule $\lim_{x\to c}\left(f(x)\right)^n = \left(\lim_{x\to c} f(x)\right)^n$

7. Root rule $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$

Use properties of limits to evaluate

Given $\lim_{x\to a} f(x) = 5$ and $\lim_{x\to a} g(x) = -3$, find each limit.

A. $\lim_{x\to a}[2f(x)]$

$2\boxed{\lim_{x\to a} f(x)}$

$2\cdot 5$

10

10. $\lim_{x\to a}[g(x)^2]$

$\left(\lim_{x\to a} g(x)\right)^2$

$(-3)^2$

$\boxed{9}$

11. $\lim_{x\to a}[f(x)g(x)]$

$\left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$

$5\cdot -3$

-15

12. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$

$\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{5}{-3}$

13. $\lim_{x\to a}[f(x)+g(x)]$

2

14. $\lim_{x\to a}[f(x)-g(x)]$

8

One Sided Limits

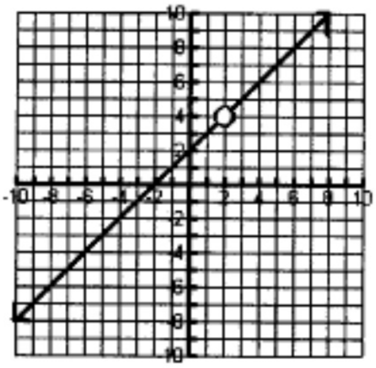
1

a) Find the $\lim_{x \to 1} f(x)$

b) Find the $\lim_{x \to 2^+} f(x)$

c) Find the $\lim_{x \to 2^-} f(x)$

d) Find the $\lim_{x \to 2} f(x)$



2. $f(x) = \begin{cases} x^2+2, & x>0 \\ 2-2x, & x<0 \\ 2, & x=0 \end{cases}$

right side of zero or 0^+

left side of zero 0^-

a) Find the $\lim_{x \to 0^+} f(x) = 2$

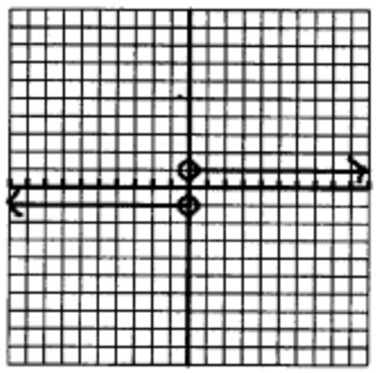
b) Find the $\lim_{x \to 0^-} f(x) = 2$

c) Find the $\lim_{x \to 0} f(x) = 2$

3) a) Find the $\lim_{x \to 0^+} f(x)$

b) Find the $\lim_{x \to 0^-} f(x)$

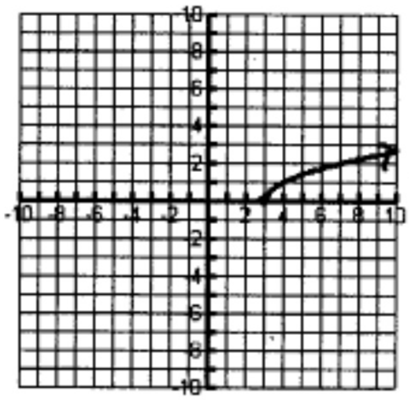
c) Find the $\lim_{x \to 0} f(x)$



4) a) Find the $\lim_{x \to 3^+} f(x)$

b) Find the $\lim_{x \to 3^-} f(x)$

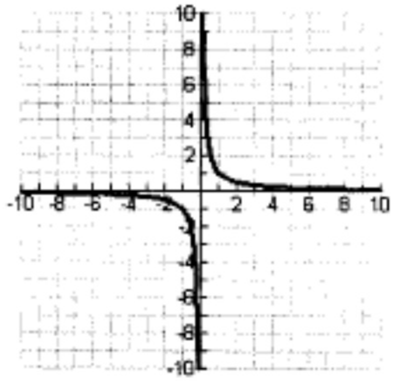
c) Find the $\lim_{x \to 3} f(x)$



5) a) Find the $\lim_{x \to 0^+} f(x)$

b) Find the $\lim_{x \to 0^-} f(x)$

c) Find the $\lim_{x \to 0} f(x)$



Infinite Limits

~ fancy math for a horizontal asymptote.

~ ∴ follow 3 rules:

1. Top heavy there isn't one; d.n.e.
2. Bottom heavy zero
3. matching ratio of the leading coefficients

Evaluate

1. $\lim\limits_{x\to\infty} \dfrac{3-x^{2}}{4+5x^{1}} = \text{dne}$

2. $\lim\limits_{x\to\infty} \dfrac{1x^{0}}{x^{1}} = 0$

3. $\lim\limits_{x\to\infty} \dfrac{5(2-x^{2})}{6x^{2}+x-1} = \dfrac{-5}{6}$

$10-5x^{2}$

4. $\lim\limits_{x\to\infty} \dfrac{4}{x-1} = 0$

5. $\lim\limits_{n\to\infty} \dfrac{n-2}{n^{2}+4} = 0$

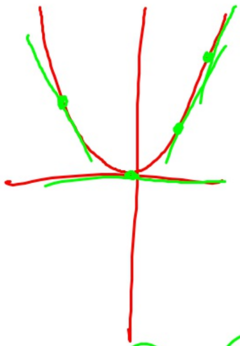
6. $\lim\limits_{n\to\infty} 4+\dfrac{1}{n^{2}} = 4+0 = 4$

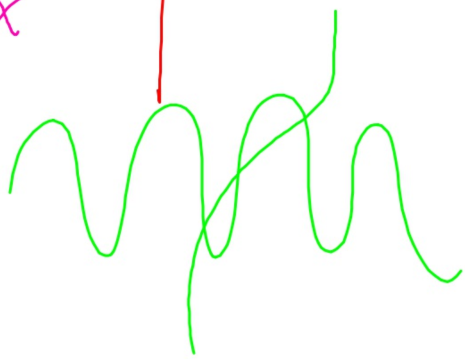
$\dfrac{n^{2}4}{n^{2}1} + \dfrac{1}{n^{2}}$

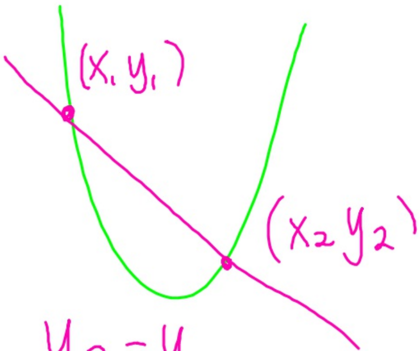
$\dfrac{4n^{2}+1}{n^{2}}$

Derivatives (Shush)

- Slope of a curve
- $f'(x)$ f prime of x
- $\frac{d}{dx}$
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$







$$\frac{y_2 - y_1}{x_2}$$

find derivative of $f(x) = x^2 + 1$

$$\lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\lim_{h \to 0} 2x + h = 2x$$

$f'(x) = 2x$