Warm up

1.Find a rectangular equation for each of the following polar equations.

a. $r=6\cos\theta+2\sin\theta$ b. $\theta=\pi/3$

2. Finda set of parametric equations for $y=2x+5$ for the given parameters:

a. $t=\sqrt{x}$ b. $t=x^3$ c. $t=\log x$

3. Find the rectangular equation by eliminating the parameter:

a. $x=t+1$ $y=t^2+3$ b. $x=2^t$ $y=t^2$

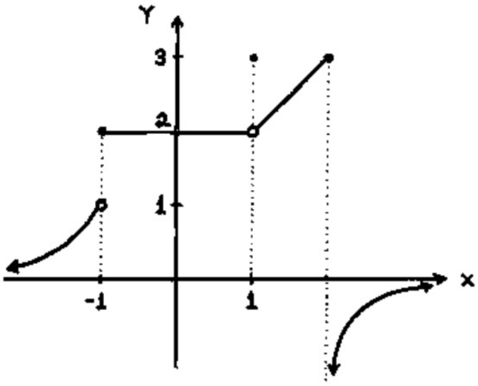
4. Graph the equation $y=2\sin 4\theta$. Give the domain, range and max and min values.

Objective: Define and find a simple limit

If f(x) becomes infinitely close to a value "L" as x approaches c from either side, the "limit of f(x) as x approaches c is L".

What is happening:

- Y-value as x gets close to c from the right
- Y value as x gets close to c from the left
- If these values match, this is the limit.



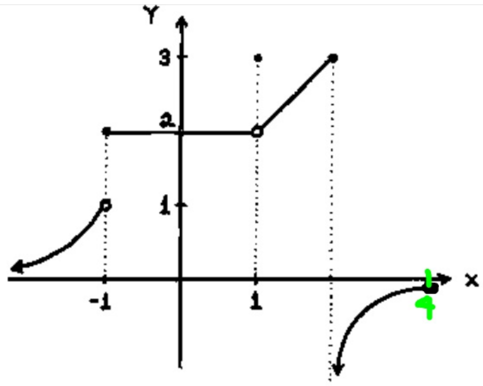
The Limit is always a number or it does not exist (DNE).

Ways for a limit to be DNE.

1) lim left ≠ lim right

*around a VA

*jump discontinuity

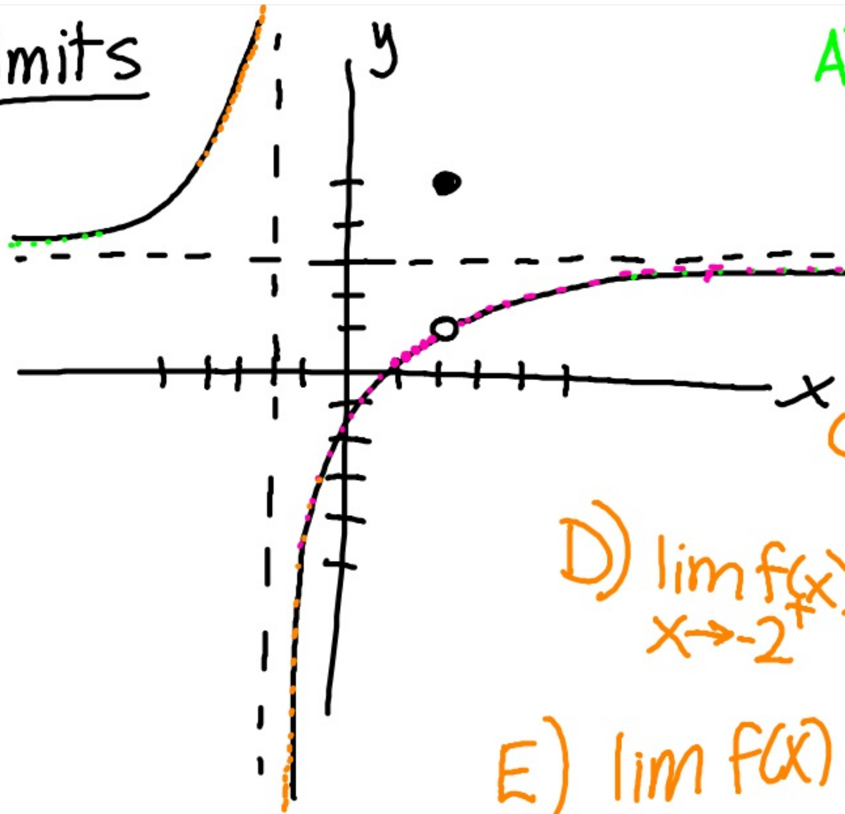


2) No graph on one side of that x value

3) VA at the x value

4) graph oscillates at that x value $y = \sin\left(\frac{1}{x}\right) \Rightarrow x = 0$

Limits



A) $\lim_{x \to \infty} f(x) = 3$

B) $\lim_{x \to -\infty} f(x) = 3$

C) $\lim_{x \to -2^-} f(x) = \infty$

D) $\lim_{x \to -2^+} f(x) = -\infty$

E) $\lim_{x \to -2} f(x) =$ dne

F) $\lim_{x \to 2^-} f(x) = 1$

G) $\lim_{x \to 2^+} f(x) = 1$

H) $\lim_{x \to 2} f(x) = 1$

I) $f(2) = 5$

How to find a limit:

1. Estimating by tables
2. Direct substitution
3. Factor and cancel
4. Rationalizing (conjugates)
5. Common denominators

Estimating by tables

$$\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = 1+1+1$$

1. $f(x) = \frac{x^3 - 1^3}{x - 1}$ Find $\lim_{x \to 1} f(x) = 3$

1. type f(x) into y=
2. TABLE SET: start at value of limit
   change by .000001
3. TABLE- look at, above and below
   to obtain answer

2. $f(x) = \frac{|x|}{x}$ Find $\lim_{x \to 0} f(x) =$ dne

A) $\lim_{x \to 0^-} f(x) = -1$

B) $\lim_{x \to 0^+} f(x) = 1$

If tables aren't allowed...like in "real" math

Direct Substitution

as long as no ÷ by zero occurs

3. Find $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x - 4} = \frac{(2)^2 - 6(2) + 8}{2 - 4}$

$= \frac{4 - 12 + 8}{-2}$

$= 0$

4. Find $\lim_{x \to -3} \frac{x^2 + 4}{x^2 - 1} = \frac{13}{8}$

5. Find $\lim_{x \to 0} \frac{\sqrt{x+1} - 2}{x + 3} = \frac{\sqrt{0+1} - 2}{0 + 3} = \frac{1-2}{3} = \frac{-1}{3}$

Find $\lim_{x \to -4} \frac{\sqrt{x+4} - 2}{x - 1} = \frac{-2}{-5} = \frac{2}{5}$

7. Find $\lim_{x \to 3} \dfrac{\frac{1}{x+3} - \frac{1}{3}}{x+5}$

8. Find $\lim_{x \to 6} \dfrac{\frac{1}{x+1} - \frac{1}{5}}{x-4}$

But what about this?!?!

1. Find $\lim_{x \to 2} \dfrac{x^2 - 6x + 8}{x^2 - 4}$

try

Factoring

1. Find $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x^2 - 4} = \frac{}{4-4} = \frac{}{0}$

$\lim_{x \to 2} \frac{(x-4)(x-2)}{(x-2)(x+2)}$ $\lim_{x \to 2} \frac{x-4}{x+2} = \frac{2-4}{2+2} = \frac{-1}{2}$

2. Find $\lim_{x \to -4} \frac{x+4}{x^2 - 16} = \frac{}{(-4)^2 - 16} = \frac{}{0}$

$\lim_{x \to -4} \frac{x+4}{(x-4)(x+4)}$

$\lim_{x \to -4} \frac{1}{x-4} = \frac{1}{-4-4} = -\frac{1}{8}$

$(a-b)(a+b)$

Rationalizing (think conjugate!)

3. Find $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$ $\frac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)}$

$\lim_{x\to 0} \frac{(x+1-1)}{x(\sqrt{x+1}+1)}$

$\lim_{x\to 0} \frac{x}{x(\sqrt{x+1}+1)}$

$\lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

4. Find $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$ $\frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)}$

$\lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)}$

$\lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

Simplifying complex fractions (GCD anyone?!?)

5. Find $\lim_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

6. Find $\lim_{x \to 4} \frac{\frac{1}{x+1} - \frac{1}{5}}{x-4}$

Wrap-up

Homework p.822 #6-18 even

WARM UP

1. $\lim_{x \to 3} 5x - 4$
2. $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$
3. $\lim_{x \to 0} \frac{\sqrt{x-1}-2}{x-5}$
4. $\lim_{x \to -2} \frac{x+2}{x^2-4}$
5. $\lim_{x \to 0} \frac{\frac{1}{x+1}-1}{x}$

6. $(-6)^{2/3} \approx 3.30$

8. $\lim_{x\to\pi} \ln\left(\sin\frac{x}{2}\right) = \ln(1) = 0 = 0$

10. $\frac{a^2-1}{a^2+1}$ (Since $a^2 + 1 > 0$ for all a, we don't have to worry about division by zero.)

12. **(a)** division by zero

(b) $\lim_{x\to 3} \frac{(x+3)(x-3)}{(x+5)(x-3)} = \lim_{x\to 3} \frac{x+3}{x+5} = \frac{6}{8} = \frac{3}{4}$

14. **(a)** division by zero

(b) $\lim_{x\to 2} \frac{(x-2)(x^2+1)}{x-2} = \lim_{x\to 2} (x^2+1) = 5$

16. **(a)** division by zero

(b) $\lim_{x\to -2} \frac{|(x+2)(x-2)|}{x+2}$. Check left- and right-hand limits.

Right: $\lim_{x\to -2^+} \frac{(-1)(x+2)(x-2)}{x+2} = \lim_{x\to -2^+}(-x+2) = 4$

Left: $\lim_{x\to -2^-} \frac{(x+2)(x-2)}{x+2} = \lim_{x\to -2^-}(x-2) = -4$

Since $4 \neq -4$, the limit does not exist.

18. **(a)** division by zero

(b) The limit does not exist.

Properties of limits

1. Sum rule

2. Difference rule

3. Product rule

4. Constant Multiple Rule

5. Quotient rule

6. Power rule

7. Root rule

Use properties of limits to evaluate

Given $\lim_{x\to a} f(x) = 5$ and $\lim_{x\to a} g(x) = -3$, find each limit.

9. $\lim_{x\to a}\left[2f(x)\right]$

10. $\lim_{x\to a}\left[g(x)^2\right]$

11. $\lim_{x\to a}\left[f(x)g(x)\right]$

12. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]$

13. $\lim_{x\to a}\left[f(x)+g(x)\right]$

14. $\lim_{x\to a}\left[f(x)-g(x)\right]$

One Sided Limits

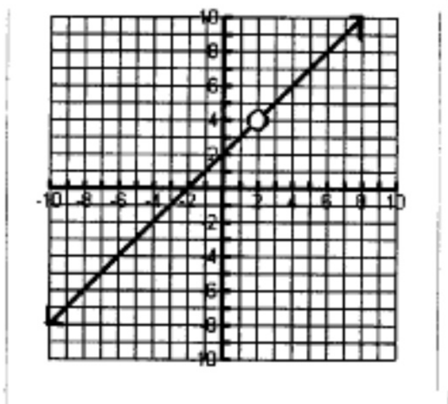
1

a) Find the $\lim_{x \to 1} f(x)$

b) Find the $\lim_{x \to 2^+} f(x)$

c) Find the $\lim_{x \to 2^-} f(x)$

d) Find the $\lim_{x \to 2} f(x)$



2. $f(x) = \begin{cases} x^2+2 & x>0 \\ 2-2x & x<0 \\ 2 & x=0 \end{cases}$

a) Find the $\lim_{x \to 0^+} f(x)$

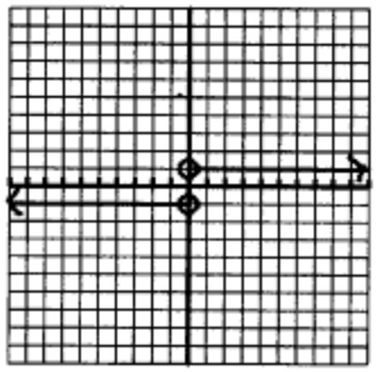
b) Find the $\lim_{x \to 0^-} f(x)$

c) Find the $\lim_{x \to 0} f(x)$

3) a) Find the $\lim_{x \to 0^+} f(x)$

b) Find the $\lim_{x \to 0^-} f(x)$

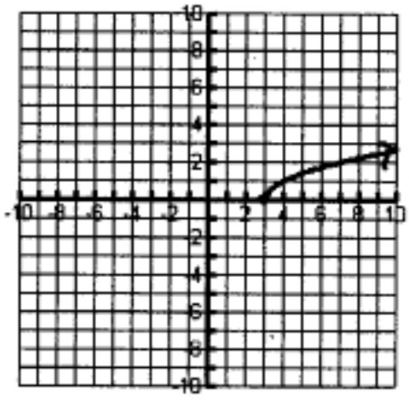
c) Find the $\lim_{x \to 0} f(x)$



4) a) Find the $\lim_{x \to 3^+} f(x)$

b) Find the $\lim_{x \to 3^-} f(x)$

c) Find the $\lim_{x \to 3} f(x)$



5) a) Find the $\lim_{x \to 0^+} f(x)$

b) Find the $\lim_{x \to 0^-} f(x)$

c) Find the $\lim_{x \to 0} f(x)$