Geometric sequences are sequences where a common ratio is multiplied to create the next term.

A geometric sequence looks like this:

55

$\cdot r \quad \cdot r$

$a_1, a_1r, a_1r^2, a_1r^3, \ldots.., a_1r^{n-1} \ldots$

Explicit formula is: $a_n = a_1 r^{n-1}$

Recursive formula is: $a_{n+1} = r a_n$

insert #

Geometric sequences multiply a common ratio to each term, creating the next term.

Notation:

a_1 means = 1^{st} term

a_n means = n^{th} term

r means = common ratio

to find the common ratio:
$\frac{a_2}{a_1}$

$$a_n = a_1 r^{n-1}$$

ex) $a_n = 4(3)^{n-1}$

$$a_n = 4\left(\frac{3^n}{3^1}\right)$$

$$a_n = \frac{4}{3}(3)^n$$

$$a_n = 4 \cdot \frac{1}{3} \cdot 3^n$$

This example shows you how a geo sequence might also be presented. It has the exponent simplified. Follow the process of changing subt to division with the same base. Then put the #'s together and leave the base to the nth power.

$$\frac{x^5}{x^3}$$

$$x^{5-3}$$

Remember your Alg1??? If same base with division, keep the base and subt. the exponents.

For each geometric sequence below

a) find the common ratio

b) find the 10th term

c) find a recursive rule for the nth term

d) find an explicit rule for the nth term

$a_{n+1} = r a_n$

$a_n = r a_{n-1}$

1. 3, 6, 12, 24, 48, ...

1 2 3 4 5

ⓐ $r = \frac{48}{24} = 2$

ⓒ $a_{n+1} = r a_n$

$a_{n+1} = 2a_n$

ⓓ $a_n = a_1 r^{n-1}$

$\boxed{a_n = 3(2)^{n-1}}$

$a_n = \frac{3}{1}\left(\frac{2^n}{2^1}\right)$

$\boxed{a_n = \frac{3}{2}(2)^n}$

ⓑ $a_{10} = 3(2)^{10-1}$

$a_{10} = 3(2)^9$

$a_{10} = 3(512)$

$a_{10} = 1536$

2. 1, -2, 4, -8, 16, ...

ⓐ $r = \frac{16}{-8} = -2$

ⓒ $a_{n+1} = -2a_n$ $a_n = -2a_{n-1}$

ⓓ $a_n = a_1 r^{n-1}$

$\boxed{a_n = 1(-2)^{n-1}}$ or $\boxed{\frac{-1}{2}(-2)^n}$

ⓔ $a_{10} = 1(-2)^{10-1}$

$a_{10} = 1(-2)^9$

$\boxed{a_{10} = -512}$

Find the nth term of the geometric sequence.

1. $a_1 = 4, r = ½, n = 10$

$a_n = 4(\frac{1}{2})^{n-1}$

$a_{10} = 4(\frac{1}{2})^9$

$a_{10} = \frac{1}{128}$

short cut

find r & mult to a_5

$-\frac{1}{3}(\frac{2}{3}) = -\frac{2}{9}$

2. $a_2 = -18, a_5 = 2/3, n = 6$

① $\begin{cases} -18 = a_1 r^{2-1} \\ \frac{2}{3} = a_1 r^{5-1} \end{cases} \Rightarrow$ ② $\begin{cases} -18 = a_1 r \\ \frac{2}{3} = a_1 r^4 \end{cases}$

Subs. to get r

③ $-18 = a_1 r$

$r = \frac{-18}{a_1}$

④ subs r's value into 2nd equation

$\frac{2}{3} = a_1\left(\frac{-18}{a_1}\right)^4$

$\frac{2}{3} = \frac{\cancel{a_1} \cdot 104976}{a_1^{\cancel{4}3}}$

$\frac{2}{3} = \frac{104976}{a_1^3}$

$a_1^3 = \frac{314928}{2}$

$\sqrt[3]{a_1^3} = \sqrt[3]{157464}$

$\boxed{a_1 = 54}$

④

$r = \frac{-18}{54}$

$r = -\frac{1}{3}$

⑤ $a_n = a_1 r^{n-1}$

$a_n = 54\left(-\frac{1}{3}\right)^{n-1}$

⑥ $a_6 = ?$

$a_6 = 54\left(-\frac{1}{3}\right)^5$

$\boxed{a_6 = -\frac{2}{9}}$

Partial Sum/Finite Sum/S_n for Geometrics

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

<u>Ex</u> $3+6+12+24$

$$S_4 = 3\left(\frac{1-2^4}{1-2}\right) = 45$$

Write the following series as a partial sum.

16, -32,

1. -2, 4, -8, ..., 64

a_1

$r = \frac{-8}{4} = -2$

$n = 6$

$$S_6 = -2\left(\frac{1-(-2)^6}{1+2}\right)$$

$$\boxed{S_6 = 42}$$

a_1 $\frac{1}{16}, \frac{1}{32},$

2. (1/2), (1/4), (1/8),......, (1/64)

$r = \frac{1}{2}$

$$S_6 = \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}}\right) = \boxed{\frac{63}{64}}$$

In Exercises 13–16, find the sum of the geometric sequence.

13. 3, 6, 12, . . . , 12,288

$\boxed{a_1}$ $\boxed{r=2}$ $S_{13} = 3\left(\frac{1-2^{13}}{1-2}\right) = 24573$

15. $42, 7, \frac{7}{6}, \ldots, 42\left(\frac{1}{6}\right)^{8}$ $50.4(1 - 6^{-9}) \approx 50.4$

$r = \frac{7}{42} = \frac{1}{6}$ $a_n = a_1 r^{n-1}$ $S_9 = 42\left(\frac{1-\left(\frac{1}{6}\right)^9}{1-\frac{1}{6}}\right)$

In Exercises 17–22, find the sum of the first n terms of the sequence. The sequence is either arithmetic or geometric.

19. $4, -2, 1, -\frac{1}{2}, \ldots; n = 12$

Infinite Sum

~ Sometimes !

$|r| < 1$

~ $S = \frac{a_1}{1-r}$

Ex Find ∞ sum, if possible.

a) 3, 6, 12, 24, ...

$r = 2$ $|r| < 1$? Nope

b) $3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

$\left|\frac{1}{3}\right| < 1$? yup!

$S = \frac{3}{1-\frac{1}{3}} = \frac{9}{2}$

EXAMPLE 4 Summing Infinite Geometric Series

Determine whether the series converges. If it converges, give the sum.

(a) $\displaystyle\sum_{k=1}^{\infty} 3(0.75)^{k-1}$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{2}\right)^{n}$ **(d)** $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots$

Homework

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