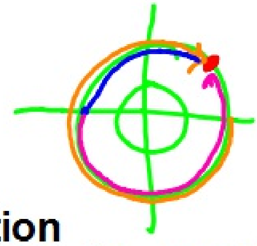
Warm up

1. Graph the polar point (2,45°)
2. Name the point in 2 other ways.



3. Remove the parameter for the equation
   x=2t+5
   y=(1/3)t+4
4. Vector v has an initial point at P(-4,5) and a terminal point at Q(-6,3). Put the vector in component form, find the magnitude, and the unit vector.

$t = \frac{x-5}{2}$ $y = \frac{1}{3}\left(\frac{x-5}{2}\right)+4$

$(2, -315°)$

$(-2, -135°)$

$(-2, 225°)$

$\frac{1}{3}\left(\frac{x-5}{2}\right)+4$

$\frac{x-5}{6}+4$

$\frac{x}{6}-\frac{5}{6}+\frac{24}{6}$

$y=\frac{x}{6}+\frac{19}{6}$

$y=mx+b$

35. $r = 3\sec\theta$

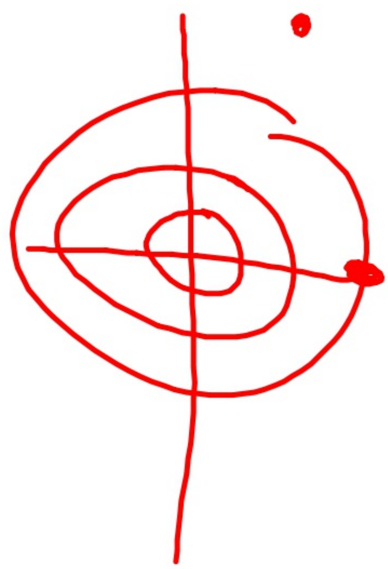
$\cos\theta \cdot r = \frac{3}{\cos\theta} \cdot \cos\theta$

$r\cos\theta = 3$

$x = 3$

$r = 3\sec\theta$

θ	r
0	3
60°	6



36. $r = -2\csc\theta$

$$\sin\theta \cdot r = -2 \, \frac{1}{\sin\theta} \cdot \sin\theta$$

$$r\sin\theta = -2$$

$$y = -2$$

39. $r\csc\theta = 1$

$\frac{1}{2}(-1)$

$\left(-\frac{1}{2}\right)^2$

$\sin\theta \cdot r \cdot \frac{1}{\sin\theta} = 1 \cdot \sin\theta$

$r \cdot r = \sin\theta \cdot r$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y + \frac{1}{4} = 0 + \frac{1}{4}$

$\frac{(x-0)^2}{\frac{1}{4}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}}$

$\frac{(x-0)^2}{\frac{1}{4}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}} =$

$\frac{(x-0)^2}{\frac{1}{4}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}}$

$\frac{(x-0)^2}{\frac{1}{4}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{4}} = 1$

38. $r \cdot r = -4\cos\theta \cdot r$

$$r^2 = -4r\cos\theta$$

$$\frac{1}{2}(4)$$

$$x^2 + y^2 = -4x$$

$$(2)^2$$

$$x^2 + 4x + 4 + y^2 = 0 + 4$$

$$(x+2)^2 + y^2 = 4$$

2) $r \cdot r = -3\sin\theta \cdot r$

$r^2 = -3r\sin\theta$ $\quad (x-h)^2 + (y-k)^2 = r^2$

$x^2 + y^2 = -3y$

$x^2 + y^2 + 3y = 0$ $\quad \frac{1}{2} \cdot 3$

$\left(\frac{3}{2}\right)^2$

$x^2 + y^2 + 3y + \frac{9}{4} = \frac{9}{4}$

$(x-0)^2 + \left(y+\frac{3}{2}\right)^2 = \frac{9}{4}$

$C\left(0, -\frac{3}{2}\right)$ $r = \frac{3}{2}$

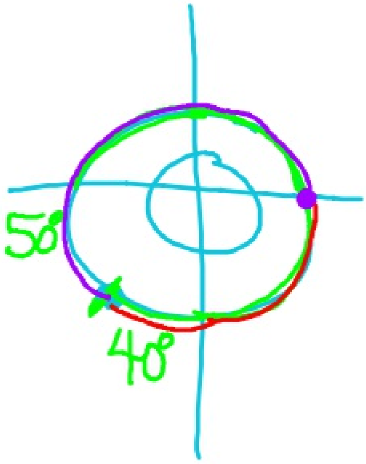
36. $r = -2\csc\theta$

$\sin\theta \cdot r = -2 \frac{1}{\sin\theta} \sin\theta$

$r\sin\theta = -2$

$y = -2$

26. $(-2.5, 50°)$ $(-2.5, -310°)$



$(2.5, 230°)$

$(2.5, -130°)$

30. $(-1,-2) \rightarrow (r,\theta)$

$x^2+y^2=r^2$

$(-1)^2+(-2)^2=r^2$

$1+4=r^2$

$r=\sqrt{5}$

$\left(\sqrt{5}, 243.4^\circ\right)$

$\left(-\sqrt{5}, 63.4^\circ\right)$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-2}{-1}$

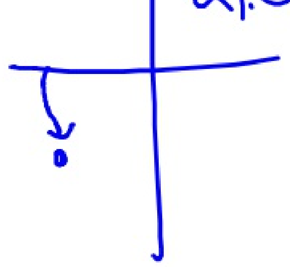
$\tan\theta = 2$

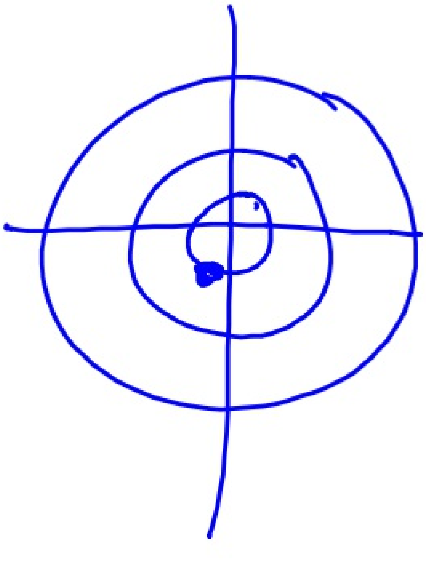
$\theta = \tan^{-1}(2)$

$\theta = 63.4^\circ$

$+180.0^\circ$

243.4°





Rectangular to Polar

$x^2+y^2=r^2$ $x=r\cos\theta$

$\tan\theta=\frac{y}{x}$ $y=r\sin\theta$

Convert rectangular to polar

1. replace all x^2+y^2 **with** r^2

2. replace x with rcosθ and y with rsinθ

3. Solve for r.

Example 1 $x^2+y^2=4$

$r^2=4$

$r=2$

$r=-2$

Example 2 $x = 5$

$$r\cos\theta = 5$$

$$r = \frac{5}{\cos\theta}$$

$$r = 5 \cdot \frac{1}{\cos\theta}$$

$$\boxed{r = 5\sec\theta}$$

Example 3 $2x+3y = 5$

$$2r\cos\theta + 3r\sin\theta = 5$$

$$r(2\cos\theta + 3\sin\theta) = 5$$

$$r = \frac{5}{2\cos\theta + 3\sin\theta}$$

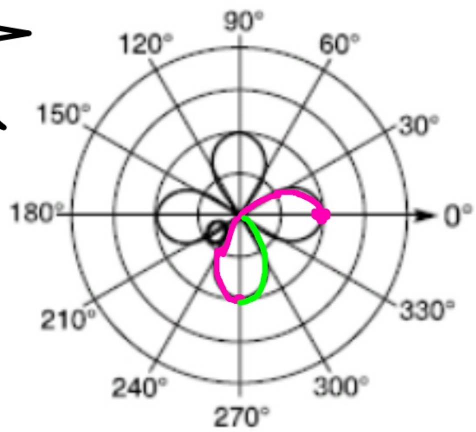
Graphing Polar Equations

A polar graph is the set of all points whose coordinates, satisfy a given polar equation.

To make a graph, use a table of values.

θ	$2 \cos 2\theta$	(r, θ)
0°	2	(2, 0°)
30°	1	(1, 30°)
45°	0	(0, 45°)
60°	−1	(−1, 60°)
90°	−2	(−2, 90°)
120°	−1	(−1, 120°)
135°	0	(0, 135°)
150°	1	(1, 150°)
180°	2	(2, 180°)
210°	1	(1, 210°)
225°	0	(0, 225°)
240°	−1	(−1, 240°)
270°	−2	(−2, 270°)
300°	−1	(−1, 300°)
315°	0	(0, 315°)
330°	1	(1, 330°)

Then plot the points on a Polar Plane



Rose Curves

equations: $r = a\ cos(n\theta)$ $r = a\ sin(n\theta)$

Characteristics:

*if *n* is even, the curve has *2n* petals.

*if *n* is odd, the curve has *n* petals.

**a* is the radius of the petals

*Involving *sine*:

*if *n* is odd, then one petal will graph on the *y*-axis in the opposite direction of the sign of *a.*

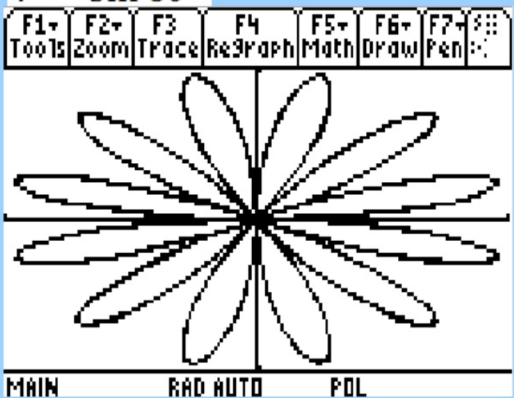
*if *n* is even, then no petal will graph on the *y*-axis.

*Involving *cosine*:

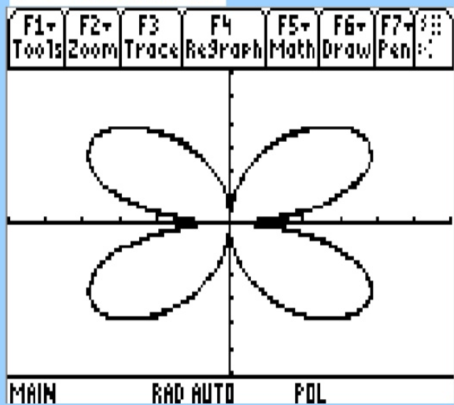
*if *n* is odd then one petal will graph on the x-axis in the direction of the sign of *a.*

*if *n* is even, then petals will graph on both the *x*- axis and the y-axis

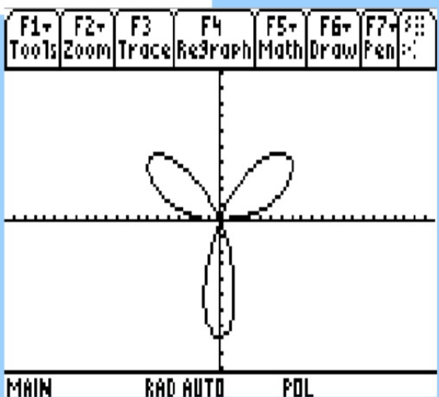
$r = \sin 6\theta$



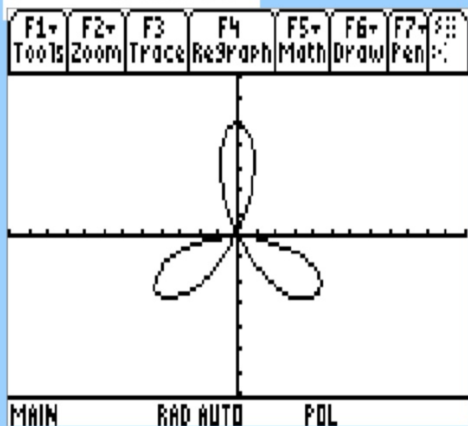
$r = 5\sin 2\theta$

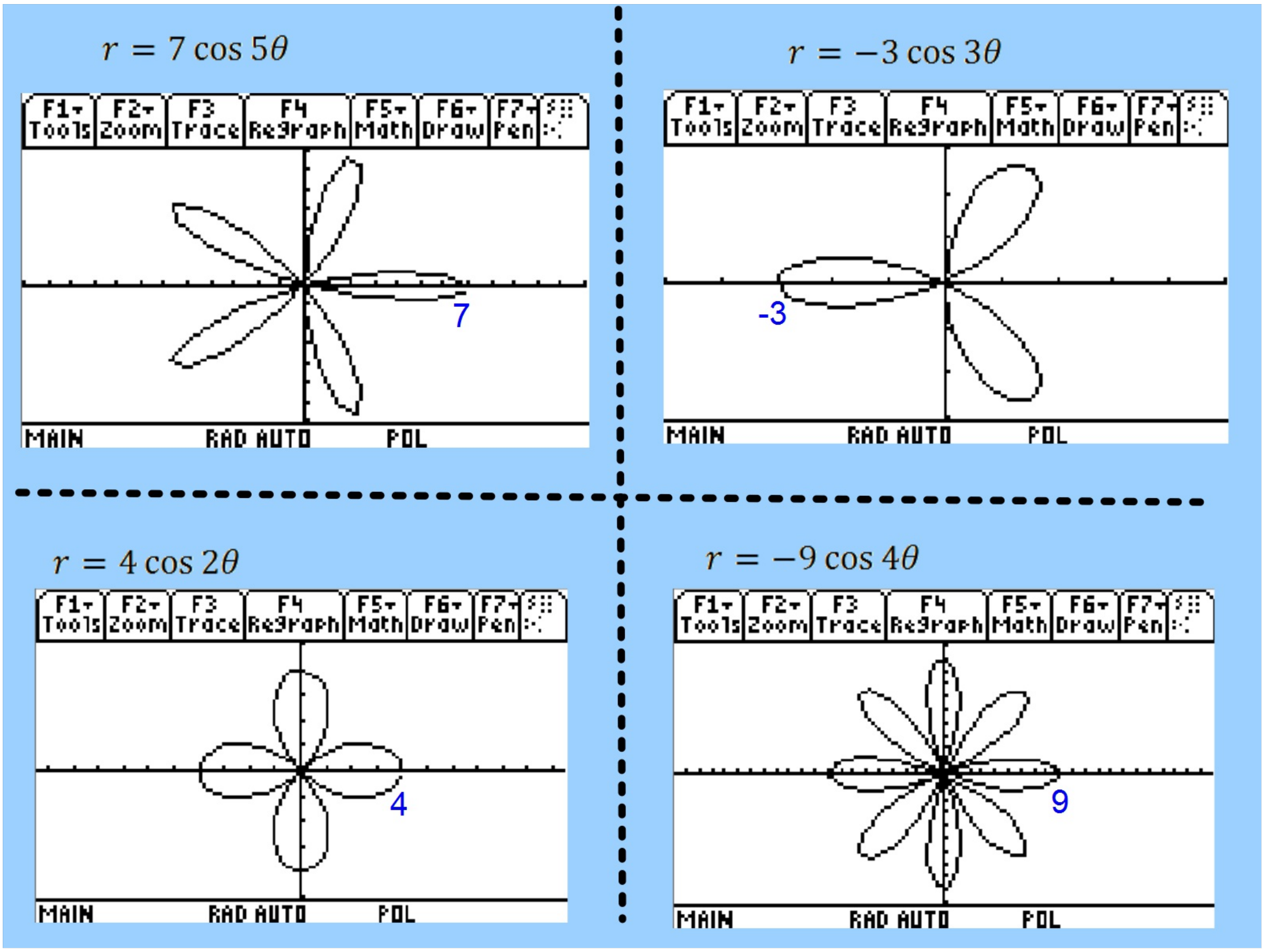


$r = 8\sin 3\theta$



$r = -5\sin 3\theta$





$r = 7\cos 5\theta$
7
$r = -3\cos 3\theta$
-3
$r = 4\cos 2\theta$
4
$r = -9\cos 4\theta$
9
F1 Tools F2 Zoom F3 Trace F4 ReGraph F5 Math F6 Draw F7 Pen
MAIN RAD AUTO POL

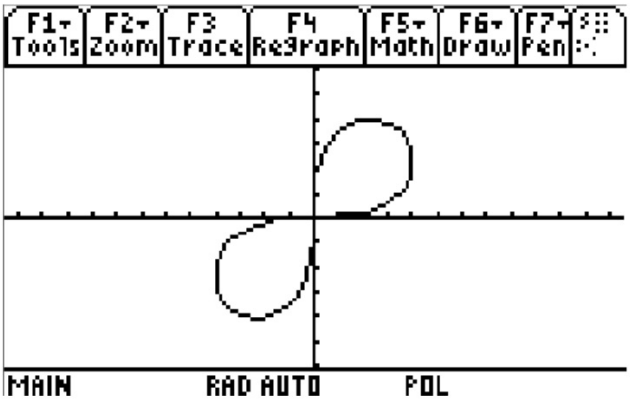
Lemniscate (tricky to enter into the calculator)

*equations: $r^2 = a^2\cos(2\theta)$ $r^2 = a^2\sin(2\theta)$

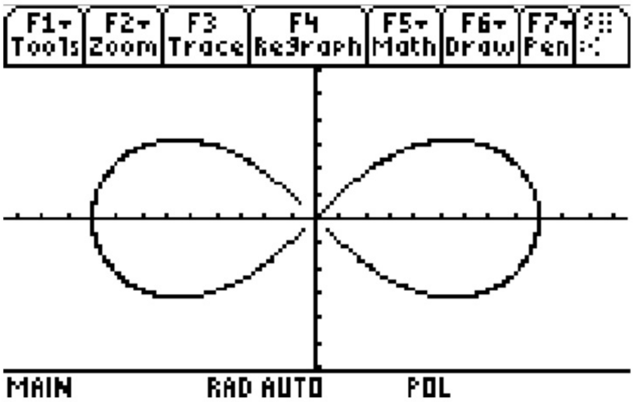
*characteristics:

* *r* is always squared
* *a* is always a perfect square
* *theta* is always doubled
* if a sine equation, then the petals will never be on an axis
* if a cosine equation, then the petals will always be on the x-axis

$r^2 = 25\sin 2\theta$



$r^2 = 81\cos 2\theta$



Limacon

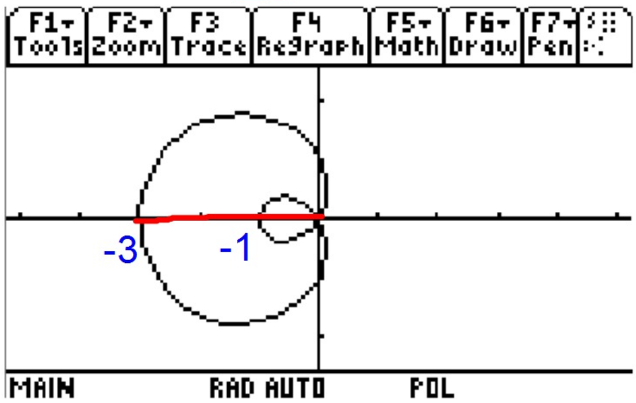
*equations: $r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$

*characteristics:

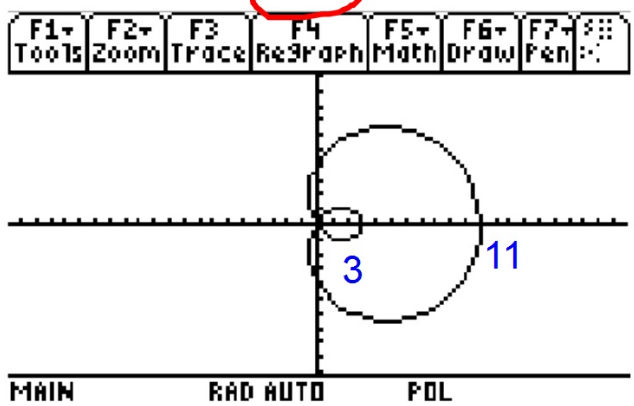
*if $a < b$, then the limacon has an inner loop. The length of the inner loop is the difference between *a* and *b*. The length of the limacon from the dimple to the end is a length of $a+b$. The dimple and circle face the left/down if $b < 0$ and face the right/up if $b > 0$. Up/Down or Right/Left is determined by sine or cosine.

*if $a > b$, we do not cover!

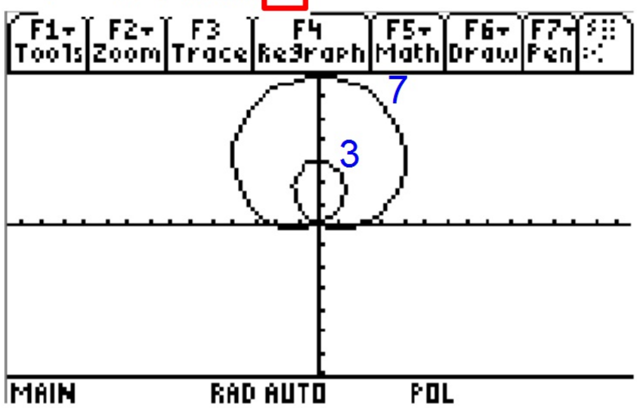
$r = 1 - 2\cos\theta$



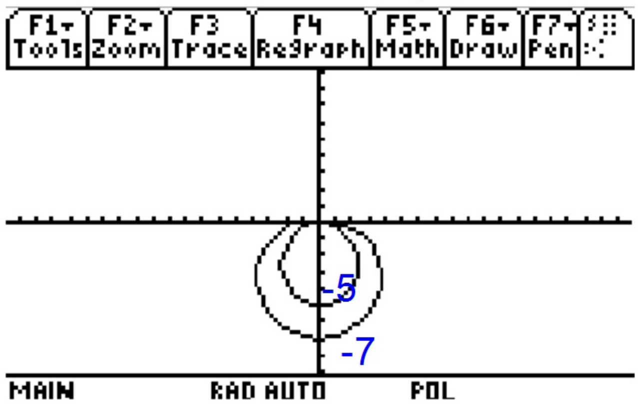
$r = 4 + 7\cos\theta$



$r = 2 + 5\sin\theta$



$r = 1 - 6\sin\theta$



Cardiods

*equations: $r = a \pm a\cos\theta$ $r = a \pm a\sin\theta$

*characteristics:

~double *a* to obtain the length between the dimple and the end

~cosine implies that the cardiod will open on the *x* axis and sine implies that it will open on the *y* axis.

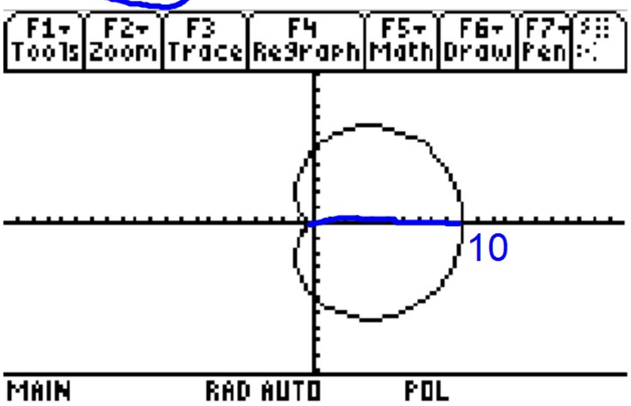
~the value of *a* is the value through which the cardiod should travel on its non-opening axis

~a positive between the constant and the trig function indicates the cardiod opens on the positive side of the indicated axis

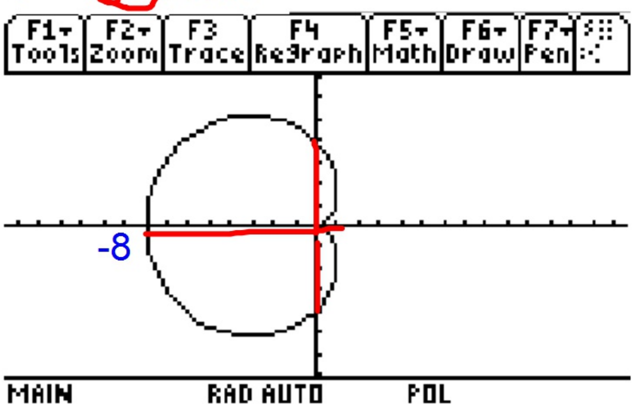
~a negative between the constant and the trig function indicates the cardiod opens on the negative side of the indicated axis.

Cardioids

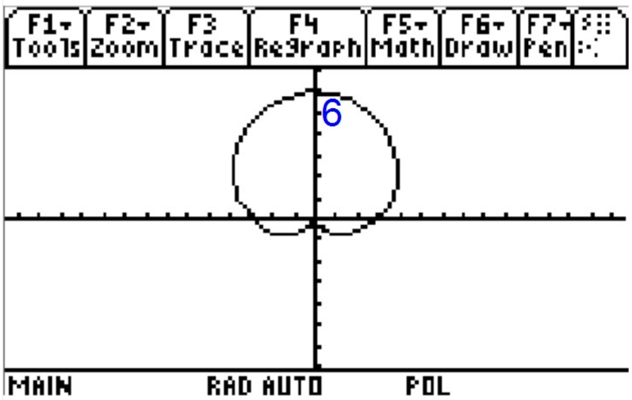
$r = 5 + 5\cos\theta$



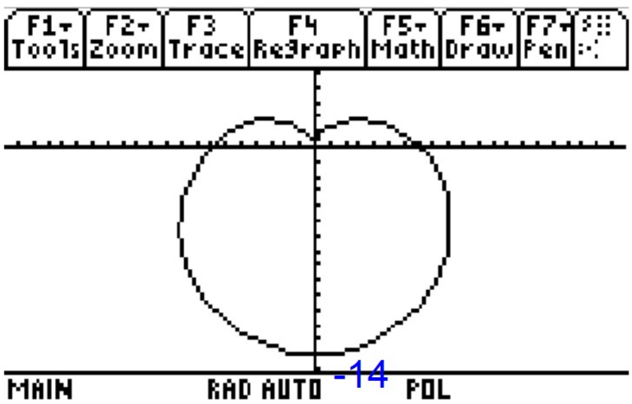
$r = 4 - 4\cos\theta$



$r = 3 + 3\sin\theta$



$r = 7 - 7\sin\theta$



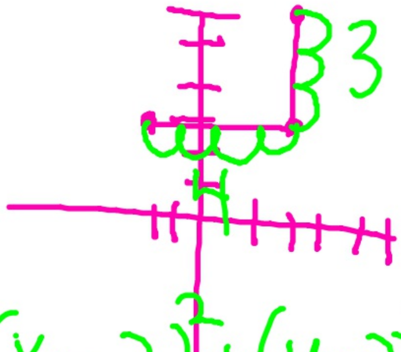
rose

limaçon

lemniscate

cardioid

Parameterization of a circle/ellipse



$x = x\text{length} \cdot \cos\theta + h$

$y = y\text{length} \cdot \sin\theta + k$

$(x-2)^2 + (y-3)^2 = 16$

C(2,3)

endpt minor (2,6)

endpt major (-2,3)

$x = 4\cos\theta + 2$

$y = 3\sin\theta + 3$

Test Tuesday:

1. draw a parametric equation from a table
2. Make a table from a parametric equation
3. Eliminate the parameter when trig present
4. Eliminate the parameter when no trig present
5. Change from Rect. to Para. given a particular parameterization.
6. convert polar point into rect. point (1 with theta in degrees and one with theta in radians)
7. convert rect. point into polar point
8. convert polar equation into rect. equation
9. convert rect. equation into polar equation
10. parameterize a circle and an ellipse
11. plot a polar point and state 3 additional names for it
12. given a rose curve equation, state the length of a petal and the number of petals.